
A Novel Hankel Singular Values-Based Order Truncation Approach Applied to SG Filter

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Abstract

A novel filter order reduction method based on Hankel singular values is proposed in this article. The weak states of a high order FIR filter having extremely low singular values are truncated to get a significantly low order IIR filter while satisfying the given frequency response specifications. The suggested method is used to design low order hardware efficient Savitzky-Golay (SG) filter for signal smoothening. The effectiveness of proposed method is verified by analyzing its denoising capability on noisy Electrocardiogram (ECG) signals. Various parameters have been used for performance evaluation. Results reveal that the suggested approach leads to a highly simplified and economic design without affecting the denoising performance of the filter. The proposed order reduction technique reduces the amount of hardware components and the amount of power consumed without compromising the integrity of the signal.

Keywords: Hankel singular values Savitzky-Golay (SG) filters ECG

1. Introduction

HE modelling procedure, quite often, leads to Tcomprehensive description of a system in the form of high order differential equations and a complex model is obtained. The increased complexity often makes it difficult to understand the behavior of a system. Thus, analysis and control of such systems is a highly challenging task. It is therefore, desirable to obtain a lower order model, which adequately reflects the dominant characteristics of original system. The reduced order system not only facilitates the theoretical understanding of a system but also leads to a simplified design in terms of hardware implementation. Thus number of components required to realize the system and power consumption are significantly reduced. Further exact analysis of high order system is both difficult and expensive. The preliminary design and optimization of such systems can frequently be achieved effortlessly, if a low order model with good approximation is derived.

In this paper a simple method is suggested to reduce high order FIR filters to lower order IIR filters while retaining the important aspects of original system [1,2,3]. The applicability of suggested method is tested to design lower order Savitzky-Golay (SG) filters. SG filters diminish the effect of noise on a signal while preserving the width and height of peaks in the signal waveform [4].

The main objective is to decrease the order of an FIR

filter (SG smoothing filter) and derive a reduced order IIR filter. FIR filter has several advantages like linear phase response, finite duration of startup transients, and continuous stability. Moreover, FIR filters satisfies the linear phase or constant group delay as compared to IIR filter for distortion-free transmission of waveforms in the passband. However, FIR filters may be practically difficult to realize because of their very high order than IIR filters for the given magnitude specifications. The lower order IIR filter provides very close approximation to magnitude response of the original FIR filter in the passband. Hence, it can be used to design reduced order SG filters which have tremendous applications in smoothing bio medical signals.

The methodology of converting FIR filter to reduced order IIR approximation can be described as: the transfer function of FIR filter is first converted to the corresponding state space representation. The Hankel matrix is then computed from state space model by evaluating Markov parameters. The Hankel singular values are calculated and analyzed. The weak states having extremely low singular values are truncated to get a new reduced order system [5,6,7,8,9]. Innumerable order reduction techniques have been implemented in different applications [10,11,12,13]

The remaining paper is arranged as follows. Section II explains the methodology of reducing FIR filter to its lower order approximation. In section III

response of reduced order SG filter is compared with its original high order filter to analyze the effectiveness of proposed method. Finally, conclusion is drawn in section IV.

2. Problem Statement And Formulation

A generic causal FIR filter of N coefficients can be represented in z-domain as follows [14]:

$$G(z) = d_0 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3} + \dots + d_k z^{-n} \quad (1)$$

where 'k' is the order of FIR filter, d_0 to d_n are complex valued filter coefficients which have imaginary as well as real values. The system of order $n=N-1$ may be expressed in state space controller canonical form as [14]:

$$A = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$C = [c_1 \quad c_2 \quad c_3 \quad \dots \quad c_n] \quad D = c_0 \quad (2)$$

where dimensions of state space matrices are $A^{n \times n}$, $B^{n \times 1}$, $C^{1 \times n}$, $D^{1 \times 1}$. Hankel matrix is computed using Markov parameters [14]:

$$G_k = CA^{k-1}B \text{ for } k = 1, 2, 3, \dots \quad (3)$$

G_k represents the Markov parameters from which Hankel matrix is derived (Equation 4).

$$H = \begin{bmatrix} H_1 & H_2 & H_3 & \dots \\ H_2 & H_3 & H_4 & \dots \\ H_3 & H_4 & H_5 & \dots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \quad (4)$$

For the given system (equation 2), Markov parameters are computed using equation (3) and the obtained Hankel matrix is given as:

$$H = \begin{bmatrix} d_1 & d_2 & \dots & d_n \\ d_2 & d_3 & \dots & 0 \\ d_3 & d_4 & \dots & 0 \\ d_n & 0 & \dots & 0 \end{bmatrix} \quad (5)$$

The Hankel singular values are then computed as:

$$H = U \Sigma U^{-1} \quad (6)$$

Where $UU^{-1} = I$, with Σ being a diagonal matrix consisting the Hankel singular values arranged from the strongest state to the weakest state.

Further a subsystem consisting of weak states with smallest singular values is eliminated from the original system resulting in a reduced order system. The truncated Hankel matrix with rank k is a truncated system defined by A_k , B_k , C_k [5][14].

Where

$$A_k = U^T(2:n, 1:k)U(1:n-1, 1:k)$$

$$B_k = U(1, 1:k)^T,$$

$$C_k = CU(1:n, 1:k), \quad (7)$$

$U(i: j, k: m)$ is extracted from matrix U 's rows i to j with columns k to m . The truncated state space system is then converted to transfer function and a reduced order IIR filter is obtained. The reduced order filter in its z-domain is represented as:

$$G_k(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_k z^{-k}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_k z^{-k}} \quad (8)$$

Where k is the reduced filter order, a_i , b_i , are the reduced order IIR filter coefficients.

The original FIR filter having transfer function $G(z)$ and the reduced order IIR filter of transfer function $G_k(z)$ are analyzed on the basis of magnitude and phase spectrum and number of hardware elements i.e. adders and multipliers used to design the filters.

3. Analysis Of Reduced Order SG Filter

A 20 order SG filter is used for analysis in this paper with a polynomial fit order of 4. The filter considered for order reduction is used for processing ECG signals in ECG monitors. Table I shows Hankel singular values for the filter under consideration. It is

observed from the results that first six singular values are dominant while the remaining are weak states and may be truncated. Hence, the filter can be reduced to an IIR filter of order approximately 6 or above. The order to which the filter can be reduced depends not only on the Hankel singular values but also on the final specifications of response. Thus, order of the reduced order filter depends on the application and may be set by the designer.

Further it is observed from the magnitude and phase response of the original FIR filter, shown in Fig. 2, that the cut off frequency is 45 Hz. Hence, reduced order IIR filter should give a response similar to original filter in the passband region i.e. 0-45Hz.

Table- I: Hankel singular values of FIR filter

Order no.	Singular value Decomposition of Hankel Singular values		
1	0.961044985804578	11	0.00588271264878976
2	0.740492045377420	12	0.00551483025624800
3	0.344489535804313	13	0.00525384285661992
4	0.171126100787533	14	0.00503534638264563
5	0.136011923388104	15	0.00487800332395742
6	0.102757504780018	16	0.00475655454146837
7	0.00891512926598427	17	0.00467582324060649
8	0.00765603499788762	18	0.00462752995742378
9	0.00694396593760113	19	0.000172591031776125
10	0.00630438332529848	20	0.00000785637692585232

It is also observed that magnitude response of 7 order IIR filter is quite similar to the response of 20 order SG filter in the passband. Further the response of reduced order system deviates from that of high order filter at frequency higher than the cut off but this does not affect the filter performance as it falls in the stop band.

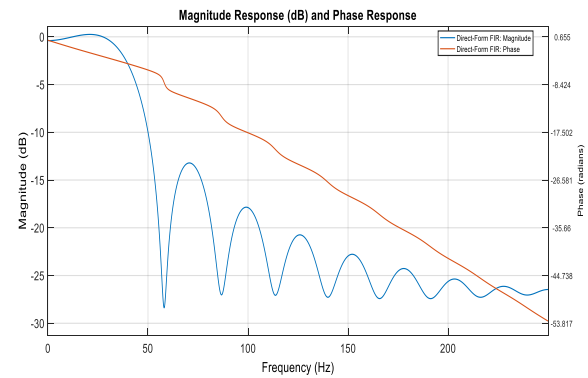


Fig. 1.The magnitude and phase response of 20 order FIR (SG) filter

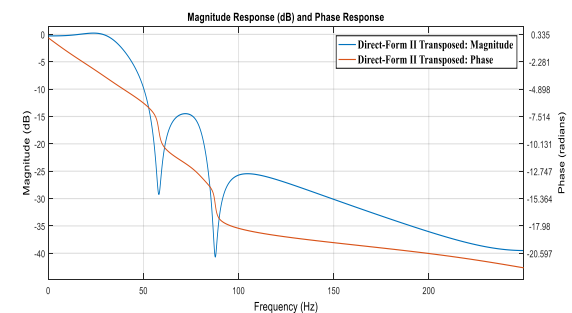


Fig. 2.(a) Magnitude and phase response of the original FIR filter

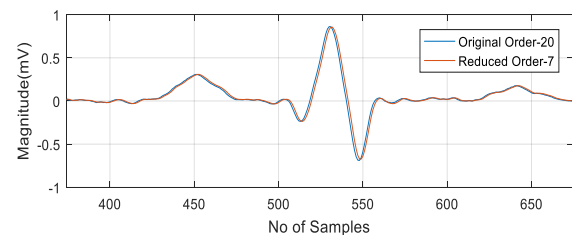


Fig. 2 (b). 7th reduced order IIR filter response

The performance of the suggested filter is analyzed on synthetic ECG [15] signal corrupted with white Gaussian noise (Fig. 3). In this work rigorous analysis is carried out to obtain the optimum order of reduced order filter. The snr obtained for different orders is evaluated and given in table 2. It is observed that 7 order filter provides same snr as 20 order filter therefore the given 20 order SG is reduced to 7 order IIR filter. The noisy ECG signal is filtered with high order SG and low order IIR filter and the results obtained are shown in Fig. 4.

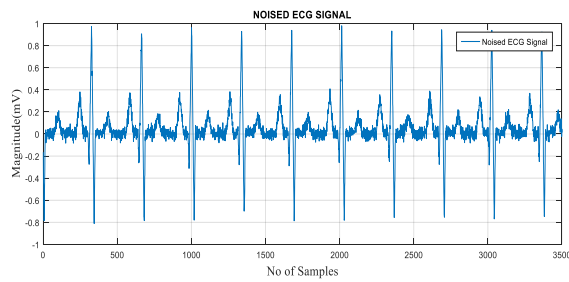


Fig. 3.Synthetic ECG signal corrupted with white Gaussian noise

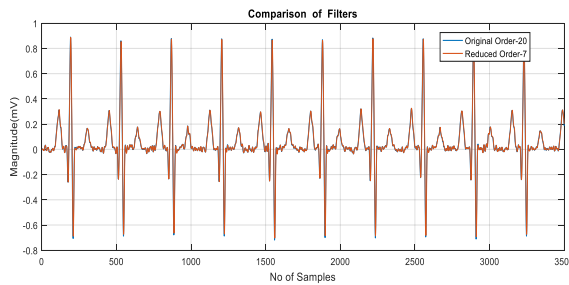


Fig. 4. Denoised ECG signal using reduced order filter and the original high order filter.

It is extracted from the results that real time response of the reduced order IIR filter follows the response of high order FIR filter very closely. Hence, information contained in the signal is not lost and original features of the signal are retained. The suggested methodology is an effective way to design a simple and economic system. Figure 5 displays the zoomed view of filtered ECG signal.

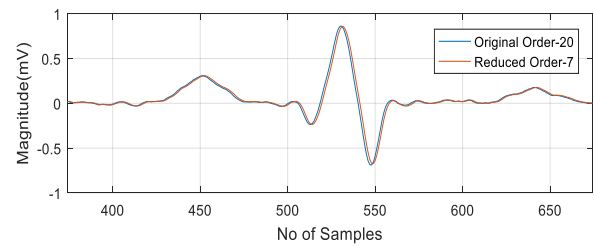


Fig. 5. The zoomed view of filtered ECG signal

4. Conclusion

In this work a 20 order SG filter is reduced to 7 order IIR filter using Hankel singular values. The obtained magnitude and phase response of the reduced order system are highly similar to the original system in passband. The designed filter is tested on ECG signal contaminated with white Gaussian noise. It is extracted from the denoising results that order reduction of filter does not affect its performance and features of the original signal are retained. The suggested methodology significantly reduces the order leading to a highly simplified design of SG filter. Hence it is concluded that suggested order reduction method reduces the number of hardware elements and power consumption without the loss of information in the signal

References

- [1] Maamar Ahfir, Izzet Kale, Daoud Berkani, "An Alternative Approach to the Balanced Model Truncation Algorithm for Acoustic Minimum-Phase Inverse Filters Order Reduction", International Scholarly Research Notices, vol. 2011, Article ID 971051, 6 pages, 2011. <https://doi.org/10.5402/2011/971051>
- [2] Zhisheng Duan, Jingxin Zhang, Cishen Zhang, Edoardo Mosca, A simple design method of reduced-order filters and its applications to multirate filter bank design, Signal Processing, Volume 86, Issue 5, 2006, Pages 1061-1075, ISSN 0165-1684, <https://doi.org/10.1016/j.sigpro.2005.07.029>.
- [3] R. W. Aldhaheri "Design of linear-phase IIR digital filters using Singular perturbational model reduction", IEE Proc.-Vsi. Image signal Process., vol. 147, no. 5, Oct. 2000.
- [4] Çağatay Candan, Hakan Inan, A unified framework for derivation and implementation of Savitzky-Golay

Table. 2. SNR Comparison for different order filters

Type	Order	Snr
Original	20	15.9734
Reduced	19	15.9734
Reduced	18	16.1384
Reduced	17	15.8927
Reduced	16	15.8828
Reduced	15	15.8262
Reduced	14	16.0733
Reduced	13	15.9472
Reduced	12	16.2404
Reduced	11	16.0919
Reduced	10	15.9447
Reduced	9	16.2655
Reduced	8	15.8584
Reduced	7	16.0472
Reduced	6	12.9567
Reduced	5	11.0012
Reduced	4	11.4015
Reduced	3	11.8800

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- filters, Signal Processing, Volume 104, 2014, Pages 203-211, 2014
- [5] R. Prasad, "Modelling and Reduction of Large Scale Systems," Department of Electrical Engineering, Indian Institute of Technology Roorkee, India, June 2004.
- [6] B. S. Chen, S. C. Peng, B. W. Chiou, "IIR filter design via optimal Hankel-norm approximation", IEE, Proceedings-G, vol. 139, no. 5, Oct. 1992.
- [7] S.C. Peng, B.S. Chen, and B.W. Chiou, "IIR filter design via optimal hankel-norm approximation," Proc. Inst. Elect. Eng., vol. 139, 1992, pp.586–590.
- [8] B. Beliczynski, I. Kale, G. D. Cain, "Approximation of FIR by IIR Digital Filters: An Algorithm Based on Balanced Model Reduction", IEEE Trans. Signal Processing, vol. 40, no. 3. Mar.1992.
- [9] K. Glover, "All optimal Hankel-norm approximations of linear multivariable systems and their L_∞ -error bounds", int. J.Control, 1984, vol. 39, no. 6, pp. 1115-1193.
- [10] Zahoor, S., Naseem, S., & Meng, W. (2017). Design and implementation of an efficient FIR digital filter. Cogent Engineering, 4(1). <https://doi.org/10.1080/23311916.2017.1323373>
- [11] Ahmad Jazlan, Umair Zulfiqar, Victor Sreeram, Deepak Kumar, Roberto Togneri, Hasan Firdaus Mohd Zaki. Frequency interval model reduction of complex fir digital filters. Numerical Algebra, Control and Optimization, 2019, 9(3): 319-326. doi: 10.3934/naco.2019021
- [12] Penumutchi, B., Maddu, K. & Kaparapu, B. Design and Application of Silicon on Insulator Based SiGe VTFET in IIR Filter by Balanced Truncation (BT) Method of Model Order Reduction. Silicon 15, 1429–1442 (2023). <https://doi.org/10.1007/s12633-022-02086-8>
- [13] Omar, A.; Shpak, D.; Agathoklis, P. Nearly Linear-Phase 2-D Recursive Digital Filters Design Using Balanced Realization Model Reduction. Signals 2023, 4, 800-815. <https://doi.org/10.3390/signals4040044>
- [14] A. Jazlan, V. Sreeram, R. Togneri, W. A. Mousa, "A Review on Reduced Order Approximation for Digital Filters with Complex Coefficients Using Model Reduction," in Australian Control Conference, Perth, Australia
- [15] Agarwal S, Rani A, Singh V, Mittal AP. Performance evaluation and implementation of FPGA based SGSF in smart diagnostic applications. Journal of medical systems. 2016 Mar 1;40(3):63.