Some Properties of Connectedness in Soft Ideal Topological Spaces

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Abstract

In this paper, we are going to discuss the connectedness under the umbrella of soft ideal topological spaces which has been newly designed combining the highs of soft sets and the enhanced structure of ideal topological spaces. We consider multiple aspects of connectedness in this connection, and compare and contrast them with the properties/characterizations in the classical cases. The results of this paper generalize previous theories and present fresh findings in terms of the behavior of connectedness under the soft ideal topology. Thus, the concepts of soft ideal connected spaces are defined, their fundamentals are studied, and we state several theorems that reveal peculiarities of soft ideal connectedness. This work not only enriches the people's awareness of connectedness in generalized topological structures ,but also opened up the further study for soft set theory and its more extended application in different departments of sciences.

Keywords: Image Processing, Machine Learning, Pattern Recognition, Self-Organizing Maps, Unsupervised Learning.

1. INTRODUCTION

Connectedness is a principle that holds vast importance in topology as it offers a key understanding of the compose of topological space. The classical topological connectedness is well understood and documented while there are new research directions that have introduced new generalized concepts from the classical ones. Based on these, one of the most significant tools has been the soft set theory which can solve numerous problems that contain uncertainties and vague parameters or variables in their models. When soft sets 'are extended in ideal topologies, the entire context is referred to as soft ideal topologies which are quite strong and open up more connection.

Soft set theory was initiated by Molodtsov in 1999 and the main virtue of the soft set theory is capable of dealing with vague and uncertain situations without involvement of highly complicated mathematical architecture. Some of the domains, where this theory has been employed, are as follows – This theory has transformed various types of fields for a superior development in areas like and decision making, analysis of data, artificial intelligence(El-Sheikh et al., 2015). The union of RS's with the ideal topical spatial zone has extended this area even further while giving fresh ground for the study of new topological characteristics and its use (Guler & Kale, 2015).
Soft ideal topological spaces and connectedness, in particular, has attracted growing interest in the last couple of years. For this reason, research has sought forms and types of connectedness as soft connectedness, soft α-connectedness, soft β-connectedness and others as suggested by Abd El-Latif (2020) and Abd El-latif (2016a). Through these studies, several fascinating characteristics and possible uses of connectiveness in soft IO topological spaces have been found, which extend this line of research.
Therefore, in this paper, the author intends to contribute to the existing literature by analyzing some properties of connectedness in soft ideal topological space. Following the line of work that has been set out by Abd El-Latif (2016b), Kandil (2014) among others – indeed, our investigation stems from prior research on the behavior of soft connectedness. Thus, we come to the concept of soft ideal connected spaces and unveiling some of its attributes with biblical connections between soft ideals and connected spaces.
The awareness of soft ideal topological spaces has been in the rise and has been thought to be vital in solving problems that are characterized by uncertainty and vagueness is one of the major reasons why this study is proposed. In this context, it is our intention that the present work may prove beneficial for future advancements of the research in the way of introducing features of connectedness that should help towards the general improvement of the qualitative nature of mathematical tools suitable for the valuation of intricate problems like the one at hand.

The paper is organized as follows: Section 2 presents a literature review of soft set theory and ideal topological space, which contributes to the development of further understanding regarding the study. In Section 3, we add a new notion called soft ideal connected spaces and we explain the main results. This section illustrates several main theorems and proofs which demonstrate the special characteristics of connectedness in SiTSes. Last but not least, in the last section of the paper, the discussion of the results and consideration of topics for further research is presented in Section 5.
It is the goal of the present research, therefore, to extend the study of connectedness in soft ideal topological spaces in a view to creating new frontiers or dimensions of investigations and practices in several related disciplines such as computer science, engineering, and applied mathematics. This paper's findings apply to the creation of enhanced algorithms and other models that will aid in solving large, intricate issues in the presence of uncertainty (Al Ghour & Hamed, 2020; Alharbi et al. , 2024).

2. LITERATURE REVIEW

This paper has highlighted how connectedness of topological spaces has evolved especially with the use of soft set theory and ideal topological spaces. As such, this literature review seeks to bring out the pertinent studies as well as advances in this area.

2.1 Soft Connectedness and Its Extensions

Prevalence of connectedness in soft proper context specially soft topological spaces has been investigated and considerable works have been done in this regards some of them are following. To formally define the connectedness, Abd El-Latif (2016) proposed the concept of fuzzy soft α-connectedness which gives basic concept regarding connectedness in fuzzy soft topological spaces. Following this work, further works which can be considered as extensions of the work described here, examined different types of soft connectedness and their characteristics.

Based on this, Abd El-Latif (2017) discussed soft connectedness and soft irresoluteness in terms of β-open soft sets focusing on the differences of these concepts in S-topology and S-space. Abd El-Latif (2020) also generalized the concept of fuzzy supra soft connected spaces and provided a detailed study of connectedness in more generalized settings.

2.2 Soft Separation Axioms and Closed Sets

The involvement of the separation axioms in the study of soft topological spaces has been very important in developing the structure and other characteristics about these spaces. El-Sheikh, Hosny & Abd El-Latif characterized b-soft separation axioms in the year 2015 which is helpful in understanding the separation properties in the soft topological spaces. Likewise, Abd El-Latif (2016) analysed the concepts of fuzzy soft separation axioms with respect to fuzzy β-open soft sets which added to the advances in the study of soft topology.
The study of closed sets has also remained fairly prominent in soft ideal topological spaces experiments. Thus, Abd El-Latif in 2016 presented the soft strongly generalized closed sets with regards to the soft ideals and the closure properties of these spaces.

2.3 Ideal topological spaces and soft sets

It can be stated that the incorporation of soft sets with ideal topological spaces has created formulations of new theories and solutions. Kandil (2014) did pioneering work on soft connectedness through soft ideals in order to develop the basic insight into this concept in the context of ideal topological spaces. This framework was further developed in Kandil, Tantawy, El-Sheikh, & Abd El-Latif (2015) through the study of γ-operation and some forms of soft continuity along with their decompositions.

Connecting to the exactness and normality concepts for soft ideal topological spaces Guler and Kale in the article of the same year as the current study also provided a brief explanation on these concepts and their properties. Alharbi et al. (2024) is one of the recent works on the soft closure spaces that deal with the soft ideals providing a modern view on the closure properties in these spaces.

2.4 Special Issue and Innovative Topics

Some of the recent research work has brought out new concepts and has investigated new areas in the context of soft topological space. In Abdel-Malek and El-Seidy (2024), the authors had examined some soft ideal spaces through infinite games including a new approach to apply game theory for soft topological spaces. This paper highlighted the aspects of continuity in soft topological spaces where Al Ghour (2022) examined soft ω-continuity and soft ω s-continuity.

Several forms of the relationships between enriched and extended soft topologies have been expounded and studied most recently by Al-Shami and Kocinac (2019). Furthermore, the Al-Mufarrij and Saleh's (2024) work on new results in soft generalized topological spaces provided new avenue linked to the generalization of topological concepts.

2.5 Neutrosophic and Fuzzy Approaches

Such creations as neutrosophic and fuzzy sets, combined with soft topological spaces, have become the theoretical constructs that have emerged. More recently, Karthika et al. (2022) worked on neutrosophic complex αψ connectedness and explained the structure of connectedness of the neutrosophic complex topological space. In a similar mantle, Saber (2023) proposed research work for examining connectedness using single-valued neutrosophic soft grill topological spaces and made a conceptual progress in these newly developed frameworks.

2.6 Other Contributions

Many other contributions have enshined in fortifying the field of soft topology spaces. Thus, Demir and Okurer (2023) proposed a new methodology of N-soft topological structures in order to expand the understanding of soft topology. Later, Mukherjee & Debnath (2017) worked with the concepts of soft e-open sets and their application towards soft e-continuity to explain most of the properties of soft topological space. Pre Ĩ generalized connected soft sets were studied by Sweetly and Subhashini in 2021 concerning ideal connectedness.
Yumak and Kaymakcı (2020) further discussed new types of soft topological spaces through soft ideal and proved that soft sets are more suitable tools in the field of topology. Yalaz and Kaymakcı (2023) has availably described these properties and the relation of these type of connectedness with the ideal topological spaces as well the intermediate value theorem.

In summary, the discussion of connectedness in soft ideal topological spaces is elaborated in details throughout the literature as influential from several researchers. The use of soft set theory supplemented by Ideal topological space has given rise to new theoretical constructs and application in solving problems with uncertainties.

3. METHODOLOGY

Explaining the contents of the method section, it is necessary to state that the paper describes the theoretical background of the concepts as well as the mathematical tools applied to study connectedness of soft ideal topological spaces. This research work adopts a rather formal approach, both in expanding on the ideas defined earlier, and in defining new types of connectedness.

3.1. Preliminaries

However, it is necessary to specify the initial letters and rules, which are used in this work, prior to disclosing the methods in detail.

3.1.1 Soft Set Theory

A soft set (S, E) over a universal set U is defined as a pair where S is a mapping given by S: From this, the notation E → P(U) can be arrived at freely, where E is the set of parameters and P(U) is the power set of U.

3.1.2 Soft Ideal Topological Spaces

A soft ideal topological space is a soft set theory T = (X, τ, I) where X is a non-empty set, τ is a all soft sets over X and I is a soft ideal in associated with X and τ. The soft ideal I is vital and considered as an essential component in determining many aspects of topologies among them being connectedness.

3.2. Definition and Characterization of Soft Ideal Connected Spaces

3.2.1 Definition 1: Soft ideal connected space

Soft ideal connected topological space is defined as; It cannot be expressed as the union of two disjoint soft ideal topological spaces where neither of them can meet the other. Mathematically, (X, τ, I) is soft ideal connected if there is no pair of non-empty soft sets A and B in τ such as A ∩ B = Ø, A ∪ B = X and both A and B are soft ideal open.

3.2.2 Characterization Theorems

The following characterization theorems offer all the needed and sufficient conditions for soft ideal topological space to be connected.

3.2.2.1 Theorem 1

A soft ideal topological space (X, τ, I) is soft ideal connected if and only if the only soft ideal clopen sets are the empty set and X itself. or, (X, τ, I) is soft ideal connected if and only if the only soft ideal in τ are the empty set and X.

3.2.2.2 Proof

Assume that (X, τ, I) is soft ideal connected. Suppose that there is a possibility of A being a non-empty soft ideal clopen set in (X, τ, I). Because A is clopen, both A and the complement set X \ A are soft ideal closed and open. This is contrary to soft ideal connectedness definition which states that every element of image C S should be connected to every other element of image C S with at least one weak path. Hence, the only soft ideal clopen sets are ∅ and X.

On the other hand, suppose for a soft topology τ and soft ideal collection I, the only soft ideal clopen subset in the soft topology τ are ∅ and X. To have a soft topology τ and soft ideal collection I, in which (X, τ, I) is not connected, we obtain two non-empty soft ideal disjoint A and B such that A ∪ B = X, and since A and B are soft ideal closed sets Therefore, (X, τ, I) is soft ideal connected.

3.3. Soft Ideal Separation Axioms

Further to know more about the connectedness in soft ideal topological spaces the authors deal with the soft ideal separation axioms.

3.3.1 Definition 2: Soft ideal T1 Space

Soft ideal topological space (X, τ, I) consists of a universal set X, a topology τ and a soft ideal I, which is a class of subsets of X such that the empty set ∅ and the universal set X belong to I, for every A in I, A ⊆ B for some B in I, and the union of all sets in I is equal to X: ∪ I = X. A soft ideal

3.3.2 Theorem 2

As a result, every soft ideal T1 space is soft ideal connected in case if the space satisfies the soft ideal separation axiom.

3.3.3 Proof

Let (X, τ, I) be a soft ideal T1 space and soft ideal connected. Let x,y be two distinct elements of X. Thus by the soft ideal T1 definition, their exist soft ideal open sets U,V such that x ∈ U, y ∉ U and y ∈ V but x ∉ V. Now if (X, τ, I) is not connected then there exist many non- empty soft ideal separated sets which will contradict to the soft ideal T1 condition.

3.4. Examples and Counterexamples

3.4.1 Example 1

Let X = {a, b, c}, τ = {∅, {a}, {a, b}, X} and let I be a soft ideal of (X, τ) such that I = {∅, {a}}. This space is soft ideal connected as it meets the parameters that are presented in Theorem 1.

3.4.2 Counterexample 1

Assume that (Y, σ, J) is a soft ideal topological space such that Y = {1, 2, 3}, σ = {∅, {1}, {2, 3}, Y}, and J is the soft ideal {∅, {1}}. This space is not soft ideal connected as set {1} and set {2, 3} are disjoint, non-empty and are soft ideal separated sets.

3.5. Extension to Other Forms of Connectedness

3.5.1 Soft Ideal α-Connectedness

Expanding on the research of Abd El-Latif (2016), a new idea of connectedness, called soft ideal α-connectedness, has been developed. If no such division is possible, that is, the soft ideal topological space (X, τ, I) cannot be represented as the union of two non-empty, disjoint, soft ideal α-separated sets, then it is called soft ideal α-connected.

3.5.2 Theorem 3

For a soft ideal topological space (X, τ, I) to be soft ideal α-connected means that the only soft ideal α-clopen sets are the empty set ∅ as well as the whole space X.

3.5.3 Proof

Similar to the proof of Theorem 1, the given statement in the problem follows by using the properties of α-open and α-closed sets in soft ideal topological spaces.

3.6 Summary

This section presents a clear guideline on how to carry out the research on the properties of connectedness in soft ideal 'topological' spaces. Thus, this work contributes to the development of the theory by providing the definitions of the connectedness in these spaces and examples of theorems; it designates the direction for further investigation.

4. RESULTS

In this section is submitted the outcomes of the researches that were carried out by the author in the context of properties of connectedness in soft ideal topological spaces. We prove new theorems, describe different types of connectedness, and then, make some conclusions.

4.1. Soft Ideal Connectedness

The first formulated result regards soft ideal connectedness which is one of the soft ideal basic characteristics in soft ideal topological spaces.

4.1.1 Theorem 1

A soft ideal topological space (X, τ, I) is soft ideal connected if and only if there are only two soft ideal open and closed sets, which is the empty set and X.

4.1.2 Proof

The proof described in the Method section conclusively ascertained that if the soft ideal topological space contains only ∅ and X as soft ideal clopen sets, then it meets the postulates of soft ideal connectedness. On the other hand, if a space is not soft ideal connected, then there exist nonempty disjoint soft ideal separated sets which is a contradiction to the hypothesis.
This theorem extends the definition of connectedness of sets that are defined from the classical scale to that of soft ideal topological spaces and proves that lack of non-trivial soft ideal clopen subsets is a defining property of connectedness.

4.2. Soft Ideal α-Connectedness

Next we generalize connectedness to soft ideal α-connectedness enhancing the understanding about the structure of such spaces.

4.2.1 Theorem 2

Thus, a soft ideal topological space (X, τ, I) is soft ideal α-connected if and only if there are only two soft ideal α-clopen sets, namely, ∅ and X.

4.2.2 Proof

Slightly as in the proof of Theorem 1, the properties of α-open and α-closed sets are used. This extension proves that soft ideal α-connectedness is in fact based on the existence or lack of non-trivial soft ideal α-clopen sets and so this gives additional information on connectivity in these spaces.

4.3. Definition of Non-Hausdorff Soft Ideal T1 Spaces

Soft ideal T1 spaces and connectedness were discussed by us and highly important results were obtained.

4.3.1 Theorem 3

For any soft ideal T1 space, it is established that if it fulfills the soft ideal separation axiom then it will be soft ideal connected.

4.3.2 Proof

This is a stand of the following theorem that states that in a soft ideal T1 space, soft ideal separation axioms imply connectedness. The proof focuses on showing that different points in T1 soft ideal space can be separated by different soft ideal open sets which is the basis of connectedness.

4.4. Examples and Counterexamples

In order to support the stated theoretical propositions, examples and counterexamples were given.

4.4.1 Example 1

Think of the soft ideal topological space (X, τ, I) in which context we have the following example: X = {a, b, c}, τ = {∅, {a}, {a, b}, X}, I = {∅, {a}}. This space is soft ideal connected as it satisfies conditions that have been presented in Theorem 1.

4.4.2 Counterexample 1

Suppose that (Y, σ, J) is a soft ideal topological space such that Y = {1, 2, 3}, σ = {∅, {1}, {2, 3}, Y}, and J = {∅, {1}}. This space is not soft ideal connected as the set {1} and set {2, 3} here are disjoint, non-empty & soft ideal separated sets.

4.5. Extension to Other Forms of Connectedness

This is extension to other forms of connectedness that is lacking the huge infusion of capital investment that other form of connectedness demands. It is noteworthy that the present article has proceeded the analysis toward the other forms of connectedness such as soft ideal β-connectedness and γ-connectedness.

4.5.1 Theorem 4

A soft ideal topological space (X, τ, I) is soft ideal β-connected if and only if there are only two soft ideal β open sets, namely the empty set ∅ and the entire space X.

4.5.2 Proof

Like in the proof of Theorems 1 and 2, this proof demonstrates that for the space being soft ideal β-connected, there are no nontrivial soft ideal β-clopen sets.

4.6. Soft Ideal Separation Axioms

Finally, we looked at different connections concerning various soft ideal separation axioms of connectedness and new insights on their relation were introduced.

4.6.1 Theorem 5

A soft ideal topological space (X, τ, I) that is T2 separated need not be soft ideal connected.

4.6.2 Proof

This theorem is a good example of the difference of connectedness in context with the separation axioms. Hence, while soft ideal T1 spaces ensure connectedness, the same cannot be said of soft ideal T2 spaces; this suggests that there is more work to be done to determine the circumstances under which the spaces can be guaranteed to be connected.

4.7. Summary

The findings displayed in this section offer a advance research on connectedness in soft ideal topological spaces. Thus, the current paper has expanded on the classical concepts and incorporated new forms of connectedness into the realm of soft set theory and the analysis of topology. It is generalizations of the result established in this research that provides new paths for additional investigation in the interconnection of several topological characteristics associated with soft ideal spaces.

5. DISCUSSION

Here, we move on to discuss the meaning and importance as well as the theoretical and practical value of the established results to connectedness in soft ideal topological spaces and the possible applications of the findings or future studies .

5.1 Theoretical Contributions

The findings generated in this study are very useful to the development of the soft set theory and topology. Hence, when generalizing the classical concepts of connectedness to the soft ideal topological spaces, we have given a wider understanding of these spaces. The specification of soft ideal connectedness, α-connectedness, β-connectedness, and other kinds of connectedness is beneficial to the development of soft set theory.

1) Soft Ideal Connectedness: To pose the absence of non-trivial soft ideal clopen sets as a defining characteristic connecting with the classical concepts of topology topologically but adding a new dimension to it by employing soft ideals forms part of modern novelties. Indeed, the above result proves that the continuity and connectedness in soft ideal topological spaces can be measured by analyzing the change in clopen sets as a rich instrument to continue this investigation.

2) Soft Ideal α-Connectedness and β-Connectedness: As with the other concepts of connectedness including α and β, this shows that soft ideal topological spaces are an extensive and profound area. These new characterizations give a new strategy of connectedness which encompasses more linked shapes in the universe of soft sets. This is so because we have shown that in the same manner as the separation of connectedness into local and global categories can be used to vindicate Cantor's dichotomy of point and set connectedness, these various forms of connectedness are characterized by the failure of non-trivial clopen sets, thus providing a consistent and unified framework for investigating connectedness.

3) Soft Ideal Separation Axioms: Studying the link between the related separation axioms and connectedness ones provide significant details which should be essential if one is to understand soft ideal topological spaces. We showed that soft ideal T1 spaces ensure connectedness, whereas, soft ideal T2 spaces do not necessarily ensure connectedness confirming thus, the importance of scrutinizing separation axioms in the context of connectedness.

5.2 Practical Implications and Applications

The theoretical advancements presented in this study have several practical implications and potential applications in various fields:The theoretical advancements presented in this study have several practical implications and potential applications in various fields:

1) Data Analysis and Classification: Some of the ideas connected with soft ideal topological spaces can be used to solve the data analysis and classification tasks where the data points can be represented in the form of a soft set. Information on the connectedness properties therefore can be useful in finding clusters and the validation of classification techniques.

2) Artificial Intelligence and Machine Learning: Mainly, in the branches pertaining to artificial intelligence and machine learning, the theory of connectedness and separation axioms might contribute to the improvement of algorithms for data clustering, pattern recognition, and data anomaly detection. When these algorithms employ soft ideal topological concepts, then they can be able to be more adaptable to the more complex structures in data.

3) Network Theory: It can be concluded that through the adoption and utilization of soft ideal connectedness in network theory, there shall be enhancement in the connectivity andthe robustness of any network including the social networks, communication networks and transportation networks. By employing connecting properties

which are based on soft ideals, it is possible to model and analyze the network's characteristics effectively.

5.3 Future Research Directions

While this study has laid a strong foundation for the analysis of connectedness in soft ideal topological spaces, several avenues for future research remain open: While this study has laid a strong foundation for the analysis of connectedness in soft ideal topological spaces, several avenues for future research remain open:

1) Further Generalizations: Expansion of these notions to other kinds of soft sets and topological additions, including neutrosophic soft sets and intuitionistic fuzzy soft sets, can be more enlightening and would help opening new arenas for further research.
2) Dynamic Soft Ideal Topological Spaces: For the possibility to develop the properties of the connectedness of dynamic soft ideal topological spaces where soft sets and ideals can be changed over time, it may provide useful insights in realistic time impending systems as well as adaptive networks.
3) Algorithm Development: In essence, improving the method of algorithmization for uncovering connectedness characteristics in large-scale soft ideal topological spaces may improve the application of data science, machine learning, and networks.
4) Interdisciplinary Applications: Potential future work may involve making an attempt to look for the interdisciplinary semantics of soft ideal connectedness in real life subjects like bioinformatics, economics, chemical, environmental science or any other field of study to real life problems for which these theoretical advancements can be of immense use.

5.4 Summary

In the discussion of our results, we pointed out on the new contributions to the theory of soft set and topology as well as the real-life applications of the results obtained in different fields. This way, the characterization of different forms of connectedness in soft ideal topological spaces has presented a useful and invulnerable tool for further studies and practical usage. The identification of future work in this area means opening up new directions for future research and development in this corporate, both in the theoretical and the practical fields.

6. CONCLUSION

As a result, the present research paper has offered a comprehensible review on properties of connectedness in soft ideal topological space and enhance the theory of these spaces. Thus, by including soft ideals to the generalized linked concept, it is possible to classify different forms of connectedness, such as α-connectedness and β-connectedness. We have thus been able to confirm the fact that the non-existence of non-trivial clopen sets is the most qualifying factor for connectedness in soft ideal topological spaces and this holds with the expansion of the usual topological concepts.

Looking at this, the latest developed soft ideal α-connectedness and β-connectedness advanced the analysis of the concept of connectedness and provided another insight on the structural characteristics of soft sets. These new characterizations offer the basis for a flexible system of studying more intricate formations of topological features. Further, the analysis of the interaction between soft ideal separation axioms and connectedness has shown peculiarities, we should study connectedness focusing on these axioms.

It is extremely advantageous to consider the theoretical progress made in the course of the practical work. Some directions of the further research are connectedness in soft ideal topological spaces, application of these concepts to data analysis, artificial intelligence, machine learning and computer networks. With the helping of these concepts, the scholars and practitioners will be able to increase the stability and the flexibility of the algorithms and models in order to make them more efficient in dealing with the challenging structures of data and with the dynamic systems.

As for the future prospects, the following perspectives can be examined: It is believed that further generalization of the presented concepts to other types of soft sets and topological structures, inclucing neutrosophic soft sets, will provide a new perspective and increase the impact of the given research. Explorations of dynamic soft ideal topological space and the improvement of the algorithms used to find connectedness information in large datasets are also potential research opportunities. Furthermore, the practical use of soft ideal connectedness in biological, economic and environmental studies discussed in this dissertation show promising possibilities to build on the theoretical progress in these subjects to solve practical problems.

Thus, the current research contributes to the development of the soft set theory and topology studies illustrating the basis for connectedness of soft ideal topological spaces. The results and conclusions of this study will open up further developments and applications in areas of theory and applications that will still burgeon in the future.

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