Thermal Instability of Chemically Reactive Couple-Stress Ferrofluids in an Anisotropic Brinkman Porous Medium

Suman Babu[1], Nisha Mary Thomas[2]
[1,2]Department of Mathematics, Christ University, Bengaluru, India
Corresponding Author:
Suman Babu
Department of Mathematics, Christ University, Bengaluru, Karnataka 560029, India
Email: sumanbabu.maths@gmail.com

ABSTRACT
The thermal instability of a chemically reactive couple-stress ferrofluid in an anisotropic Brinkman porous medium is examined in the current work. The fluid is subjected to a tiny perturbation in the study and it is assumed that this perturbation results in a chemical reaction with zero-order energy release. While maintaining a constant temperature at the lower boundary, the system is cooled from the top layer. The Rayleigh number is determined through the application of linear stability analysis and the Galerkin method. Attention is focused on the effect of magnetic, chemical, anisotropic and couple-stress parameters on the onset of ferroconvection in an anisotropic Brinkman porous medium with chemical reaction and anisotropic properties. It is shown that the system is strongly stabilized through the presence of the porous matrix and couple stresses. The mutually antagonistic influence of the thermal and mechanical anisotropic parameters is also validated. Notably, it is substantiated that in the presence of couple stresses, the destabilizing impact of both magnetic stresses and chemical reaction could be effectively controlled. The findings of the problem shed some light on the potential use of ferromagnetic fluids in the effective control of heat transfer mechanisms.

Keywords: - Ferroconvection, Anisotropic Porous Medium, Couple Stresses, Chemical reaction, Galerkin method

1. INTRODUCTION

Convection is initiated by the interaction of the fluid’s thermal and mechanical sources through the induction of applied temperature gradients from outside. The interaction between the thermal and mechanical sources in the fluid by the induction of externally applied temperature gradients helps to kick-start convection. Lord Rayleigh [1] was the person who initially looked into the convective stability theory of a parallel fluid layer. According to the traditional Rayleigh-Bénard problem, temperature-dependent density fluctuation results in a specific transit of fluid particles at which convective instability occurs, leading to the emergence of Bénard cells, which are hexagonal convection cells. Traditional stability theory examines the Rayleigh number's critical value and distinguishes a zone of steadiness from the fluctuating state. Chandrasekar [2] provided a more complete explanation of the thermal instability of a fluid layer in various situations using the assumption of invariant density (Boussinesq approximation). Finlayson [3] developed a new path in ferroconvection by taking into account all Rayleigh-Bénard Convection (RBC) presumptions. According to him, a unique fluctuation in magnetization that is thought to be a linear function of magnetic field and temperature is the cause of magnetic convection. The first study of ferrofluids was done in the work by Papell [4]. Ferrofluids are artificially created substances that do not exist in nature in their free state. Rosensweig [5] initiated a thorough and well-organized investigation on the area of ferrofluids. Over the past three decades, significant effort has been made to investigate ferroconvection leading to a wide range of applications of magnetic fluids such as dynamic sealing, heat dissipation, damping, doping of technical materials, and so on [6], [7]. The stability of ferrofluids, which results in the usage of ferrofluids in material study, the medical field, and many technical scenarios, is affected by the presence of a vertical magnetic field [8]–[10]. Recently, Madhusree and Sameer [11] extensively studied the engineering applications ferrofluids.The consistency of RBC in a saturated porous medium, described by Horton and Rogers [12], provides a deadline for the occurrence of convection. Although the RBC problem has been extensively studied for Newtonian fluids, only very few studies have been focused on non-Newtonian fluids with convection of RBC type. The influence of couple stresses on the convective stability of a magnetized ferrofluid saturating a porous medium is investigated by Thakur et al. [13] for various bounding surface combinations. This study demonstrates that fluid confined within a rigid-rigid bounding surface exhibits enhanced thermal stability, rendering it well-suited for convective processes in ferrofluids. The results on non-Newtonian fluids are helpful in the modern period because of their growing significance.Stokes [14] developed the fundamental equations for couple-stress fluids, which are the simplest for microfluids.

The study of convective diffusion is often closely linked with chemical processes, as chemical reactions can drive convection. When the density of the product differs from the reactant, an isothermal reaction can induce free convection. Additionally, heat effects from reactions can act as a distributed heat source or sink, causing convection. Zero-order reactions, where the reaction rate is independent of reactant concentration and equal to the rate constant, also play a role. Exothermic reactions in fluid-saturated porous media generate heat, altering fluid density and triggering free convection. Kordylewski and Krajewski [15] were among the first to study this effect, followed by researchers like Farr et al. [16], Vilijoen and Hlavacek [17], and Subramanian and Balakotaiah [18], who examined the stability of free convection in porous media with pressure-dependent energy-releasing chemical reactions. Malashetty [19] investigated the early onset of free convection under isothermal and adiabatic boundary conditions based on the same assumptions adopted in the works mentioned above [15]–[18]. Nisha and Maruthamanikandan [20]–[22] extended this research to chemically reactive couple-stress ferrofluids in saturated porous media, considering the impact of steady heat flux at the lower boundary. Their findings show that magnetic and chemical effects promote the initiation of magnetic convection with zero-order energy release. Further extensions of ferroconvection have been explored by authors like Soya et al. [23], who addressed radiative heat transfer in a Brinkman porous medium containing ferrofluids.

Modeling geothermal reservoirs, thermal insulation systems, packed-bed catalytic reactors, and heat storage devices requires consideration of thermally driven convection in porous media. Traditionally, isotropic porous media have been the focus of many theoretical and experimental studies. However, in practical applications, the mechanical and thermal properties of the porous matrix are often anisotropic, such as in loft insulation, synthetic porous materials in chemical manufacturing, and coating materials. Anisotropic porous media, characterized by periodic layering in permeability and thermal conductivity, are particularly relevant in geological contexts like sedimentary rocks. Kvernvold and Tyvand [24] studied thermal convection in anisotropic porous media pertinent to insulation systems. Degan et al. [25] examined natural convective heat transmission, highlighting that maximum heat transfer occurs when principal permeability directions align with the coordinate axes, with greater permeability in the flow direction. Mahajan and Hemant [26] analyzed thermal instability in a rotating anisotropic porous layer, finding that increased thermal anisotropy and rotation parameters stabilized convection. Straughan [27] investigated the effects of anisotropic permeability and varying boundary conditions on the onset of penetrative convection in porous media. Naseer and Maruthamanikandan [28] detailed the initiation of ferroconvection in porous media, considering the combined influence of viscoelasticity, thermal anisotropy, and mechanical dislocation. Numerous additional studies have explored convection in fluid layers within anisotropic permeable media [29]–[31]. The works mentioned in the literature [32]–[45] had a significant impact on understanding the nature of the reported work.

In magnetic fluid technologies, controlling convection is crucial yet challenging. Despite extensive research, no study has yet analyzed anisotropic Brinkman ferroconvective instability caused by the interplay of magnetic and couple stresses with a non-autocatalytic exothermic reaction. Using the Darcy-Brinkman model with anisotropic permeability to describe porous media flow, and incorporating thermal anisotropy in the energy equation, this study employs stability analysis with normal modes and the higher-order Galerkin method. It derives the media Rayleigh number as a function of both magnetic and non-magnetic parameters. This research aims to understand how chemical reactions, anisotropic characteristics, and couple stresses impact the stability criteria for ferroconvection onset. The findings could benefit magnetic fluid technologies like magnetic field sensors, modulators, ferromagnetic resonators, and optical switches. Section 2 details the problem statement, basic equations, and the fluid's quiescent state. Section 3 presents the linear stability analysis, followed by the solution method in Section 4. Results are discussed graphically in Section 5, with conclusions in Section 6.

2. MATHEMATICAL FRAMEWORK

The current problem involves an unbounded, horizontally oriented ferrofluid layer with couple stresses, contained within two parallel plates of thickness *d*. The porous layer is subjected to cooling from the upper region with a temperature of T_c. A zeroth-order reaction could be activated as the temperature experiences slight variations across the entire domain from T_c. The temperature at the lower surface is denoted as T_h with T_h being greater than T_c. Gravity is assumed to act vertically downwards in this context (see Figure 1).

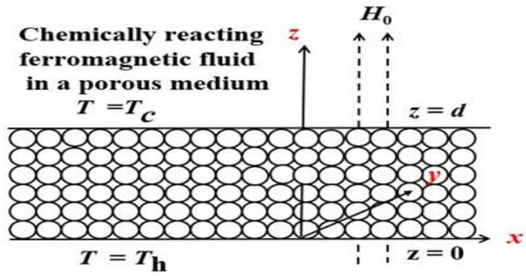

Figure 1. Physical Outline

The pertinent fundamental equations for the chemically reactive couple-stress magnetic fluid (CSMF) within a saturated porous medium, considering the Boussinesq approximation and a temperature range below the Curie point, are as follows:

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$\rho_0 \left[\frac{1}{\varepsilon} \frac{\partial \vec{q}}{\partial t} + \frac{1}{\varepsilon^2} (\vec{q} \cdot \nabla) \vec{q} \right] = -\nabla p + \rho \vec{g} + \nabla \cdot \left(\vec{H}\vec{B} \right) + \mu_f \nabla^2 \vec{q} - \mu_c \nabla^4 \vec{q} - \mu_f \vec{K} \cdot \vec{q} \tag{2}$$

$$\vec{K} = K_x^{-1} \left(\hat{i}\hat{i} + \hat{j}\hat{j} \right) + K_z^{-1} \left(\hat{k}\hat{k} \right) \tag{3}$$

$$\varepsilon c_f \left[\frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T \right] + (1-\varepsilon)(\rho_0 C)_s \frac{\partial T}{\partial t} + \mu_0 T \left(\frac{\partial \vec{M}}{\partial T} \right)_{V,H} \cdot \left[\frac{\partial \vec{H}}{\partial t} + (\vec{q} \cdot \nabla) \vec{H} \right] = \nabla \cdot \left(\vec{K}_T \nabla T \right) + Q e^{\left(\frac{-E}{RT} \right)} \tag{4}$$

$$\vec{K}_T = K_{T_x} \left(\hat{i}\hat{i} + \hat{j}\hat{j} \right) + K_{T_z} \left(\hat{k}\hat{k} \right) \tag{5}$$

$$\rho = \rho_0 \left[1 - \alpha (T - T_c) \right] \tag{6}$$

$$\vec{M} = \frac{\vec{H}}{H} M(H,T) \tag{7}$$

$$M = M_0 + \chi (H - H_0) - K_m (T - T_c) \tag{8}$$

The chemical reaction effect is introduced by the term $Qe^{\left(\frac{-E}{RT}\right)}$, where the product of the reactant concentration, a pre-exponential factor and heat of reaction, denoted as Q; the activation energy, represented by E and the universal gas constant, denoted as R. Further, T is the temperature, α is the thermal expansion coefficient, χ is the magnetic susceptibility, K_m is the pyromagnetic coefficient and $C_{V,H}$ is the specific heat at constant magnetic field and volume. The notations $\vec{K}$ and $\vec{K}_T$ refer to the anisotropic permeability tensor and anisotropic thermal conductivity tensor respectively.The pertinent Maxwells equations are given by

$$\nabla\cdot\vec{B}=0,\ \nabla\times\vec{H}=\vec{0} \tag{9}$$

$$\vec{B}=\mu_0\left(\vec{M}+\vec{H}\right) \tag{10}$$

The temperature is made dimensionless by setting $\theta=\frac{T-T_c}{T_r}$, where $T_r=\frac{RT_c^{\,2}}{E}$ with $\frac{RT_c}{E}<<1$ and the dimensionless temperature boundary conditions aligned with the objective include

$$\theta=0 \quad \text{at} \quad z=1 \quad \text{and} \quad \theta=\theta_h \ \text{at}\ z=0. \tag{11}$$

The fluid is quiescent in the initial state and is provided by $\vec{q}=\vec{q}_b(z)=0$, $\rho=\rho_b(z)$, $p=p_b(z)$, $\theta=\theta_b(z)$, $\vec{H}=\vec{H}_b(z)$, $\vec{M}=\vec{M}_b(z)$, $\vec{B}=\vec{B}_b(z)$.

The solution for the steady state from equations (1) to (10) is provided by

$$\frac{dp_b}{dz}=-\rho_b g+B_b\frac{dH_b}{dz} \tag{12}$$

$$K\frac{d^2\theta_b}{dz^2}+\frac{Qe^{\left(\frac{-E}{RT_c}\right)}}{T_r}C_f e^{\theta_b}=0 \tag{13}$$

$$\rho_b=\rho_0\left[1-\alpha T_r\theta_b\right] \tag{14}$$

$$\frac{d}{dz}\left(M_b+H_b\right)=0 \tag{15}$$

$$M_b(z)=M_0+\chi\left(H_b-H_0\right)-K_mT_r\theta_b \tag{16}$$

$$H_b(z)=H_0+\frac{K_mT_r\theta_b}{1+\chi} \tag{17}$$

$$M_b(z)=M_0-\frac{K_mT_r\theta_b}{1+\chi} \tag{18}$$

$$B_b(z)=\mu_0\left(M_0+H_0\right) \tag{19}$$

Adopting the methodology of Malashetty et al. [19], the solution of eqn. (13) reads

$$\theta_b=\ln\left(\frac{l_1}{2F}\right)+\ln\left[1-\left(\frac{1-l_2\exp\left(-\sqrt{l_1}\ z\right)}{1+l_2\exp\left(-\sqrt{l_1}\ z\right)}\right)^2\right], \tag{20}$$

where the constants l_1 and l_2 in eqn. (20) are implicitly calculated using

$$\exp\left(\sqrt{l_1}\right)\left[\frac{1-\sqrt{1-\frac{2F}{l_1}}}{1+\sqrt{1-\frac{2F}{l_1}}}\right] = \left[\frac{1-\sqrt{1-\frac{2F\exp\left(\theta_h\right)}{l_1}}}{1+\sqrt{1-\frac{2F\exp\left(\theta_h\right)}{l_1}}}\right] \quad (21)$$

$$l_2 = \exp\left(\sqrt{l_1}\right)\left[\frac{1-\sqrt{1-\frac{2F}{l_1}}}{1+\sqrt{1-\frac{2F}{l_1}}}\right] \quad (22)$$

Where $F = \frac{Cd^2}{\kappa}$ is the Frank-Kamenetskii number, $C = \frac{Qe^{\left(\frac{-E}{RT_c}\right)}}{T_r}$ and $\kappa = \frac{KT_z}{\varepsilon C_f}$.

3. Stability Analysis

Employing normal modes in the conventional stability analysis [2], [12], [19], the resulting non-dimensional equations are:

$$\frac{\sigma}{P_r}\left(D^2-k_h{}^2\right)W + R_a k_h{}^2\theta + D_a{}^2\left(\frac{1}{\xi}D^2-k_h{}^2\right)W - \Lambda\left(D^2-k_h{}^2\right)^2 W$$
$$+\Gamma\left(D^2-k_h{}^2\right)^3 W + Nk_h{}^2\left(\frac{d\theta_b}{dz}\right)D\Phi - Nk_h{}^2\left(\frac{d\theta_b}{dz}\right)\theta = 0 \quad (23)$$

$$\lambda\sigma\theta - \left(D^2-k_h{}^2\eta\right)\theta + \left(\frac{d\theta_b}{dz}\right)W - Fe^{\theta_b}\theta = 0 \quad (24)$$

$$D^2\Phi - M_3 k_h{}^2\Phi - D\theta = 0 \quad (25)$$

3.1. Stationary Instability

In light of the fact that oscillatory instability is not present, the set of equations pertaining to stationary instability is as follows:

$$R_a k_h{}^2\theta + D_a{}^2\left(\frac{1}{\xi}D^2-k_h{}^2\right)W - \Lambda\left(D^2-k_h{}^2\right)^2 W$$
$$+\Gamma\left(D^2-k_h{}^2\right)^3 W + Nk_h{}^2\left(\frac{d\theta_b}{dz}\right)D\Phi - Nk_h{}^2\left(\frac{d\theta_b}{dz}\right)\theta = 0 \quad (26)$$

$$\left(D^2-k_h{}^2\eta\right)\theta - \left(\frac{d\theta_b}{dz}\right)W + Fe^{\theta_b}\theta = 0 \quad (27)$$

$$D^2\Phi - M_3 k_h{}^2\Phi - D\theta = 0 \quad (28)$$

The applicable conditions at the boundary are

$$W = D^2W = D^4W = 0 \quad at \quad z=0 \text{ and } at \ z=1$$
$$\theta = D\Phi = 0 \qquad at \quad z=0 \text{ and } \ at \ z=1 \quad (29)$$

The set of equations (26) through (28) along with the conditions at the boundary (29) constitutes an eigenvalue problem with R_a representing the eigenvalue.

4. METHOD OF SOLUTION

Due to the variable coefficients in the boundary value problem described by equations (26) through (28), obtaining an analytical solution becomes challenging. Therefore, an approximate solution is pursued using the Galerkin method [46]. We choose suitable trial functions (simple polynomials, Legendre functions, Chebychev polynomials and the like) for the vertical component of velocity, temperature perturbation and magnetic potential that satisfy the prescribed boundary conditions, but may not exactly satisfy the differential equations. This results in residuals when the trial functions are substituted into the differential equations. The Galerkin method warrants the residuals be orthogonal to each trial function. It is known that the Galerkin method yields an eigenvalue which is stationary to small changes in the trial functions because the Galerkin method is equivalent to an adjoint variational principle. It is worth mentioning that a fourth order Galerkin method yields more accurate results with superior and desired convergence level.

5. RESULTS AND DISCUSSIONS

The current study deals with the problem of chemical reaction-driven ferroconvective instability in an anisotropic Brinkman porous medium taking into account the influence of the couple-stress effect. Due to the absence of oscillatory instability and the impracticality of obtaining an analytical solution, the Galerkin technique is exploited to compute the corresponding numerical solution. Figure 2 displays the temperature profile for different values of F concerning the quiescent state. The fluid's reaction is self-sustaining, and the lower boundary's nature shifts to adiabatic past a specific value of F [19]. Our analysis focuses solely on F below this critical value. The critical threshold of F is determined to be 0.878455 for $\theta_h = 1.19$. Moreover, it is made sure that the range of various parameters arising in the study is experimentally relevant [3], [19], [20], [47]. It is evident that an increase in F leads to heightened imbalance and deviation from linearity in the temperature profile. This distinct characteristic of the temperature pattern at the quiescent state should be considered when referring to equation (20). In terms of the physics involved, the noted escalation in imbalance and deviation from linearity in the temperature profile at the quiescent state might be associated with the chemical reaction, leading to the amplification of the heat generation rate.

Figures 3 through 5 illustrate the influence of porous media and couple stresses on the system's stability. It is apparent that the parameter of interest, the critical media-Rayleigh number, denoted by R_{ac} , escalates alongside the D_a , Λ and Γ , indicating the postponement of the ferroconvection threshold. Consequently, the presence of the porous layer and couple stress contribute to the stabilization of the system. This particular influence of porous and couple-stress parameters is in conformity with that mentioned in the existing works [13], [47].

On the other hand, Figure 6 illustrates the impact of M_3 on ferroconvection in a porous medium induced by chemical reaction and couple stresses. The parameter M_3 symbolizes the deviation toward nonlinearity in magnetization. The destabilizing impact of M_3 is evident from Figure 6 and this destabilizing influence of M_3 is relatively insignificant when the couple stresses are present. However, it is plausible that the augmentation of convection due to chemical reaction can occur in a ferromagnetic fluid due to a substantial increase in the nonlinearity of magnetization [20], [21].

Figure 2. Basic Temperature profile

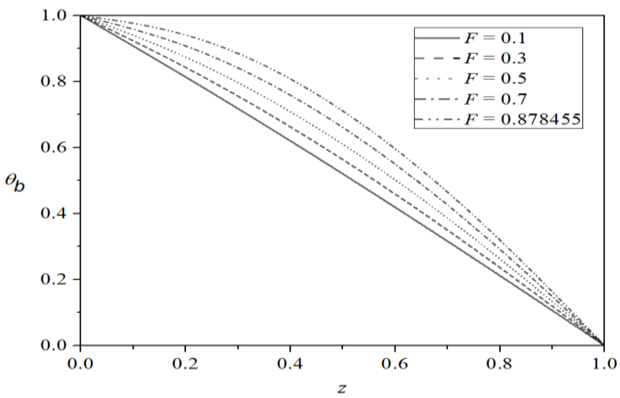

Figure 3. Graph of R_{ac} versus F

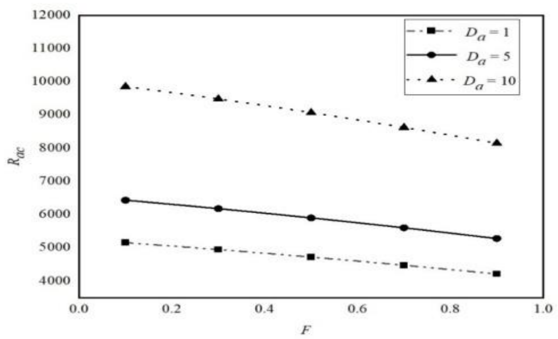

Figure 4. Graph of R_{ac} versus F

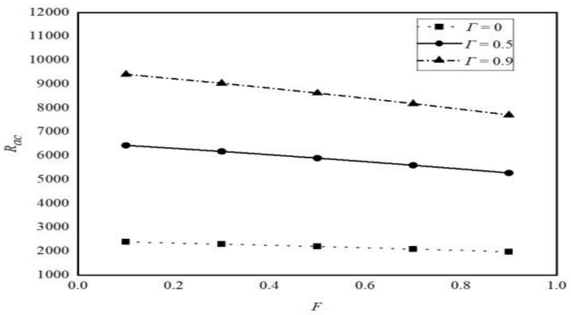

Figure 5. Graph of R_{ac} versus F

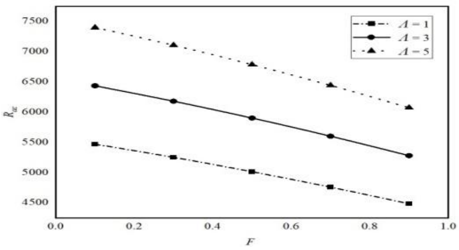

Figure 6. Graph of R_{ac} versus F

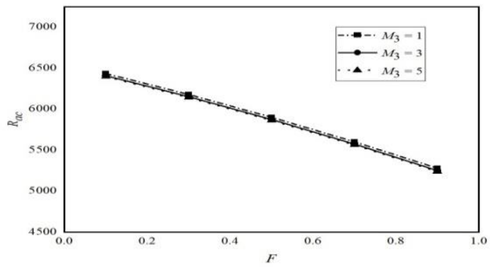

Figure 7. Graph of R_{ac} versus F

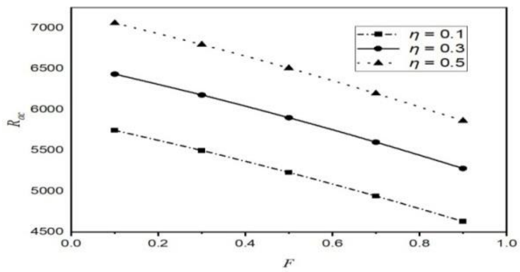

Figure 7 illustrates the impact of η on ferroconvection in a porous medium induced by chemical reaction and couple stresses. The parameter η symbolizes the deviation toward thermal anisotropy. Figure 7 clearly indicates the stabilizing impact of η on the system.Figure 8 illustrates the impact ξ of on ferroconvection in a porous medium induced by chemical reaction and couple stresses. The parameter ξ symbolizes the deviation toward mechanical anisotropy. Figure 8 clearly indicates the destabilizing impact of ξ on the system.Figure 9 portrays the combined influence of magnetic stresses and chemical reaction. Figure 9 reveals that R_{ac} decreases with the simultaneous increase in both F and N Consequently, the threshold of ferroconvection is hastened by the stresses from both magnetization and chemical reaction mechanisms. Notably, the fluctuations induced by the magnetic mechanism gains prominence solely if the F parameter reaches a significant magnitude. In the limiting scenario of Γ, F and N approaching zero, the well-known values of $a_c = \pi$ and $R_{ac} = 4\pi^2$ can be obtained as documented in the literature [32].

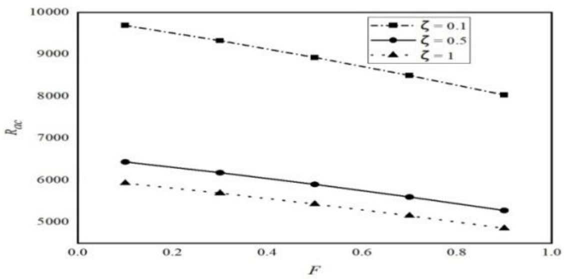

Figure 8. Graph of R_{ac} versus F

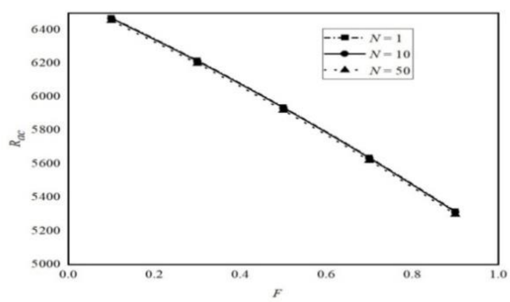

Figure 9. Graph of R_{ac} versus F

6. CONCLUSION

The investigation of ferroconvective instability of the Bénard-Brinkman type induced by the interplay of magnetic and convective forces, influenced by a non-autocatalytic exothermic reaction and couple stresses, is conducted through the stability analysis involving infinitesimally small disturbances utilizing normal modes, subsequent to the application of the higher-order Galerkin method. The analysis of the current study leads to the subsequent conclusions:

- The destabilizing effect of the chemical reaction can be largely attributed to the nonlinearity and asymmetry inherent in the thermal profiles of the basic state.
- Magnetic forces and chemical reactions both contribute to destabilizing dynamics, and they vie with each other to amplify the magnitude of instability.
- The stability of the system is strengthened by the interaction between couple stresses and porous characteristics.
- The acceleration of the convection threshold is attributed to the nonlinearity of magnetization, which diminishes in significance as the magnetization parameter M_3 reaches a significant magnitude.
- The mechanical anisotropy parameter destabilizes the system and The thermal anisotropy parameter stabilizes the system.

The findings of this study could significantly impact engineering applications related to heat transfer, not to mention those utilizing microfluidic devices.

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