Nonzero θ_{13} with $A_4 \times Z_4$ Flavor Symmetry Group

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Abstract

A neutrino mass model based on an $A_4 \times Z_4$ symmetry group is proposed to explain current neutrino oscillation data. The model has the same field contents as the original A_4 model proposed by Altarelli and Fergulio. Minimal modification to the original model is made by considering the antisymmetric contribution from A_4. The resultant mass model can give the deviation from tribimaximal mixing ($\theta_{13} = 0$) with $\theta_{23} = 45°$ in a normal hierarchy. The model also exploits the validity for both normal and inverted mass hierarchies.

Keywords: Majorana phase, discrete symmetry, mass square differences, CP-violating phase, neutrinoless double beta decay
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1. INTRODUCTION

Currently, the neutrino oscillations experiments [1, 2, 3, 4, 5, 6, 7] have measured the oscillation parameters, namely, mass squared differences (Δm^2_{21} and Δm^2_{31}) and mixing angles (θ_{12}, θ_{23}, and θ_{13}) to good accuracy. These oscillation data reveal a certain pattern behind lepton mixings. Several discrete symmetry groups like A_4, S_4, A_5, etc., are studied to explain flavor symmetry and lepton mixing [8, 9, 10]. Out of these groups, the A_4 symmetry group is the most economical. The A_4 neutrino mass model proposed by Altarelli and Fergulio [8, 11] can accommodate tribimaximal mixing (TBM) ($\sin\theta_{12} = 1/3$, $\sin\theta_{23} = 1/2$, and $\sin\theta_{13} = 0$) that was considered a good description of the neutrino mixing matrix in the past decades. In this minimal model, the three left-handed Standard Model (SM) lepton doublets (l) are fitted into triplet representation (3) of A_4 while the three generations of right-handed SM lepton singlet (e^c, μ^c, and τ^c) are assigned to three irreducible singlet representations $(1, 1', 1'')$ of A_4. In addition to these SM particles, a right-handed A_4 triplet neutrino field ν^c, two A_4 triplet flavon fields ϕ_S and ϕ_T and a singlet ξ are introduced along with two A_4 invariant Higgs doublets H_d and H_u. The right-handed neutrino field contributes to neutrino masses through a type-I seesaw mechanism. The resultant mass matrix can naturally lead to TBM.

However, the recent neutrino oscillation data indicate the small but nonzero value of θ_{13}. As a result, models based on A_4 are modified to accommodate nonzero θ_{13} and current neutrino oscillation data. These corrections are made either by introducing contributions from charged lepton sector [12, 13, 14] or including contributions to the neutrino sector from additional flavon fields [15, 16, 17] or considering the vacuum alignment corrections of the flavon fields [18, 19], etc. Interestingly, minimal modification to the original Altarelli and Fergulio (AF) model can also explain current neutrino oscillation data. In those cases, corrections to the neutrino mixing parameters are produced by considering the additional Dirac mass term arising from the antisymmetric part allowed by the A_4 symmetry along with nontrivial contribution from Majorana mass matrix [20] or trivial contribution from Majorana mass matrix with contribution from charged lepton sector as in [12].

In this work, we proposed a minimal approach to modify the original AF model to explain current neutrino data. The model has the same field content as the AF model [11]. Here, the A_4 symmetry is supplemented by the Z_4 group to obtain the desired pattern of the charged lepton mass matrix and neutrino mass matrix. The antisymmetric part of the Dirac mass term allowed by the A_4 group is considered along with other Dirac mass terms permitted by the underlying symmetry to generate nonzero θ_{13}. Unlike the earlier model [12, 20], the contribution from both the Majorana mass matrix and charged lepton mass matrix to lepton mixings is trivial as a consequence of $A_4 \times Z_4$ symmetry in our case. The model predicts some interesting results. One of the most important features is that the value of θ_{23} is fixed at 45° in the normal hierarchy (NH) as in TBM. More interestingly, the model disfavours inverted hierarchy (IH) as the Dirac CP phase (δ) predicted by our model violates the current neutrino oscillation global fit data. Further, we investigate the correlation between neutrino oscillation parameters and neutrinoless double beta decay parameters.

The paper is organized as follows. In Section 2, we describe the model along with the particle contents. In Section 3, we present the numerical analysis and the results in terms of correlation plots. Section 4 deals with the conclusion.

2. THE MODEL

The fields content in the model are similar to the original AF model. In the AF model, the contribution from Dirac mass term to neutrino mass is trivial. However, in our case, the Dirac mass terms have a contribution from the A_4 triplet flavon field ϕ_S (both symmetric and antisymmetric) and singlet field ξ due to $A_4 \times Z_4$ symmetry. Here, it is important to note that the Z_4 charge considered in the model is in additive notation. The charged lepton mass matrix is diagonal (T-diagonal) while the Majorana mass matrix has a trivial mass structure. The transformation properties of the fields use in the model are given in Table 1.

The Yukawa Lagrangian for leptons which are invariant under the $A_4 \times Z_4$ transformation is given in the following equa-

Fields	l	e^c	μ^c	τ^c	ν^c	$H_{u,d}$	ϕ_S	ϕ_T	ξ
A_4	3	1	$1^{''}$	$1^{'}$	3	1	3	3	1
Z_4	1	1	1	1	0	0	-1	-2	-1
$SU(2)_L$	2	1	1	1	1	2	1	1	1

TABLE 1: Transformation properties of various fields under $A_4 \times Z_3 \times Z_2 \times SU(2)_L$ group.

tion:

$$\begin{aligned} L_l = {} & \frac{Y_e}{\Lambda}(l\phi_T)_1 H_d e^c + \frac{Y_\mu}{\Lambda}(l\phi_T)_{1'} H_d \mu^c \\ & + \frac{Y_\tau}{\Lambda}(l\phi_T)_{1''} H_d \tau^c + \frac{y_1}{\Lambda}\xi\,(lH_u\nu^c)_1 \\ & + \frac{y_a}{\Lambda}\phi_S\,(lH_u\nu^c)_A + \frac{y_b}{\Lambda}\phi_S\,(lH_u\nu^c)_S + \frac{1}{2}M\,(\nu^c\nu^c) + h.c. \end{aligned} \tag{1}$$

The vacuum alignment for the triplet flavons is assumed as $\langle\phi_S\rangle = (1,1,1)v_S$ and $\langle\phi_T\rangle = (1,0,0)v_T$. This vacuum alignment satisfies the minimization condition of the scalar potential for the whole range of parameter space [21]. The vacuum expectation values (VEV) for the singlet flavon ξ and two A_4 invariant Higgs doublets $(H_{u,d})$ are considered in v_ξ and $v_{u,d}$ direction, respectively. After realizing the VEV, we can calculate the mass matrix of charged lepton and neutrino after flavor and electroweak symmetry breaking. Then, the charged lepton mass matrix is given by

$$M_l = \frac{v_d v_T}{\Lambda}\begin{pmatrix} Y_e & 0 & 0 \\ 0 & Y_\mu & 0 \\ 0 & 0 & Y_\tau \end{pmatrix}. \tag{2}$$

The Dirac mass has the form

$$M_D = \begin{pmatrix} 2a+c & -a+b & -a-b \\ -a-b & 2a & -a+b+c \\ -a+b & -a-b+c & 2a \end{pmatrix}, \tag{3}$$

where $a = \frac{y_b . v_u . v_s}{\Lambda}$, $b = \frac{y_a . v_u . v_s}{\Lambda}$, and $c = \frac{y_1 . v_u . v_\xi}{\Lambda}$. Majorana mass matrix takes the structure

$$M_R = \begin{pmatrix} M & 0 & 0 \\ 0 & 0 & M \\ 0 & M & 0 \end{pmatrix}. \tag{4}$$

The neutrino obtained masses through Type-I seesaw mechanism. The effective neutrino mass matrix can be derived from

$$m_\nu = \left(M_D^T M_R^{-1} M_D\right) \tag{5}$$

$$= \frac{1}{M}\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}, \tag{6}$$

where

$$\begin{aligned} m_{11} &= \frac{1}{M}\left(6a^2 - 2b^2 + 4ac + c^2\right), \\ m_{12} = m_{21} &= \frac{1}{M}\left(-3a^2 + 6ab + b^2 - 2ac\right), \\ m_{13} = m_{31} &= \frac{1}{M}\left(-3a^2 + b^2 - 2a(3b + c)\right), \\ m_{22} &= \frac{1}{M}\left(-3a^2 - 6ab + b^2 + 4ac\right), \\ m_{23} = m_{32} &= \frac{1}{M}\left(6a^2 - 2b^2 - 2ac + c^2\right), \\ m_{33} &= \frac{1}{M}\left(-3a^2 + 6ab + b^2 + 4ac\right). \end{aligned}$$

3. RESULTS

To realize the smallness of the neutrino masses, the Majorana masses are considered as $M \sim \mathcal{O}(10^{11}\,\mathrm{GeV})$. Since the neutrino mass matrix m_ν given in (6) is obtained in the basis where charged lepton mass matrix is diagonal, it can be diagonalized by unitary matrix U as $m_\nu = U^* m_{\mathrm{diag}} U^\dagger$ [17]. Further, we varied the values for model parameters a, b, and c to fix the neutrino oscillations parameters ($\Delta m^2_{21}, \Delta m^2_{31}, \theta_{12}, \theta_{23}, \theta_{13}$, and δ) to their experimental ranges. The experimental data used for the comparison are given in Table 2. The region in the parameters space that can satisfy the current oscillation data is very narrow as shown in Figures 1 and 2.

Parameters	Best fit $\pm 1\sigma$	2σ	3σ
$\theta_{12}/°$	34.3 ± 1.0	32.3–36.4	31.4–37.4
$\theta_{13}/°$(NO)	$8.53^{+0.13}_{-0.12}$	8.27–8.79	8.20–8.97
$\theta_{13}/°$(IO)	$8.58^{+0.12}_{-0.14}$	8.30–8.83	8.17–8.96
$\theta_{23}/°$(NO)	49.26 ± 0.79	47.37–50.71	41.20–51.33
$\theta_{23}/°$(IO)	$49.46^{+0.60}_{-0.97}$	47.35–50.67	41.16–51.25
$\Delta m^2_{21}[10^{-5}eV^2]$	$7.50^{+0.22}_{-0.20}$	7.12–7.93	6.94–8.14
$\|\Delta m^2_{31}\|[10^{-3}eV^2]$(NO)	$2.55^{+0.02}_{-0.03}$	2.49–2.60	2.47–2.63
$\|\Delta m^2_{31}\|[10^{-3}eV^2]$(IO)	$2.45^{+0.02}_{-0.03}$	2.39–2.50	2.37–2.53
$\delta/°$(NO)	194^{+24}_{-22}	152–255	128–359
$\delta/°$(IO)	284^{+26}_{-28}	226–332	200–353

TABLE 2: The global fit result for neutrino oscillation parameters [22].

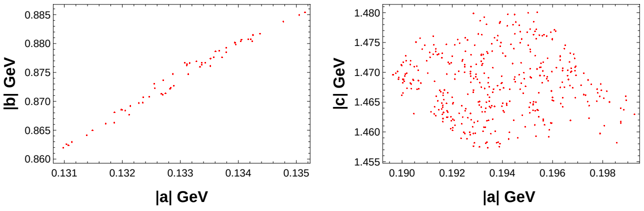

FIGURE 1: Correlation plots between the model parameters for normal hierarchy (NH).

The six observables (two mass squared differences, three mixing angles, and CP-violating phase) can be reproduced by

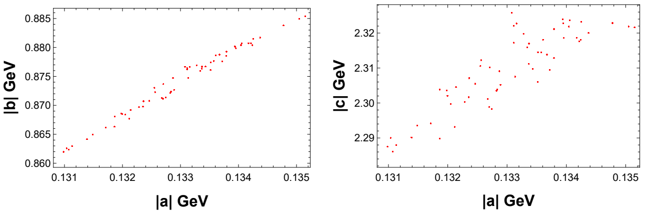

FIGURE 2: Correlation plots between the model parameters for inverted hierarchy (IH).

the three model parameters. The model predictions for the neutrino oscillation parameters for NH and IH are shown in the correlation plots Figures 3 and 4, respectively. In the case of NH, the model predicts that the values of atmospheric mixing angle θ_{23} are similar to the prediction of TBM (i.e., 45°). This predicted result of θ_{23} shows deviation from the best fit of the global fit data but lies well within the 3σ range. The values for Δm^2_{21}, Δm^2_{31}, and θ_{13} are distributed evenly throughout the 3σ range while the predicted values for θ_{12} and δ have a more definitive range. In the case of IH, our model can predict the values of mass square differences and three mixing angles in the 3σ range as shown in Figure 4. The sum of the absolute neutrino masses predicted by the model for both NH and IH is shown in Figure 5, and they are in good agreement with the current data (i.e., $<0.12\,\text{eV}$ ($0.15\,\text{eV}$) for NH (IH) [22]). However, the model prediction of $\delta = 0°$ shows deviation from current neutrino oscillation data. Therefore, our minimally modified AF model can reproduce deviation from TBM and also hints that the NH is the preferred pattern for neutrino masses.

The neutrinoless double beta ($0\nu\beta\beta$)-decay is characterized by effective Majorana mass $|m_{\beta\beta}|$.

$$m_{\beta\beta} = \left|U_{ei}^2 m_i\right| = \left|c_{12}^2 c_{13}^2 m_1 e^{i\alpha_M} + s_{12}^2 c_{13}^2 m_2 e^{i\beta_M} + s_{13}^2 m_3 e^{i2\delta}\right|, \tag{7}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ and $i, j = 1, 2, 3$ while α_M, β_M, and m_i are the Majorana phases and masses, respectively. Currently, the experimental sensitivity of $|m_{\beta\beta}|$ is obtained from Gerda [23] with an upper limit on the Majorana mass $|m_{\beta\beta}| <$ (104–228) meV corresponding to ^{76}Ge ($T_{1/2}^{0\nu\beta\beta} > 9 \times 10^{25}$ yr), CUORE [24] with an upper limit on the Majorana mass $|m_{\beta\beta}| <$ (75–350) meV corresponding to ^{130}Te ($T_{1/2}^{0\nu\beta\beta} > 3.2 \times 10^{25}$ yr), and KamLAND-Zen [25] with an upper limit on the Majorana mass $|m_{\beta\beta}| <$ (61–165) meV corresponding to ^{136}Xe ($T_{1/2}^{0\nu\beta\beta} > 1.07 \times 10^{25}$ yr). The Jarlskog invariant is given by the phase redefinition invariant quantity,

$$J = \text{Im}\left\{U_{e1}U_{\mu 2}U_{e2}^{*}U_{\mu 1}^{*}\right\} = s_{12}c_{12}s_{23}c_{23}c_{13}^2 s_{13}\sin\delta. \tag{8}$$

Figure 6 shows correlation plots between model predictions for (J) and $|m_{\beta\beta}|$ and Dirac CP-violating phase versus $|m_{\beta\beta}|$ for NH while Figure 7 shows correlation plots between model predictions for (J) and $|m_{\beta\beta}|$ for IH. The model predictions for $|m_{\beta\beta}|$ for both NH and IH are in a very narrow range. The predicted range for $|m_{\beta\beta}|$ for NH can be tested by next-to-next generation ton-scale $0\nu\beta\beta$-decay experiments [25, 26].

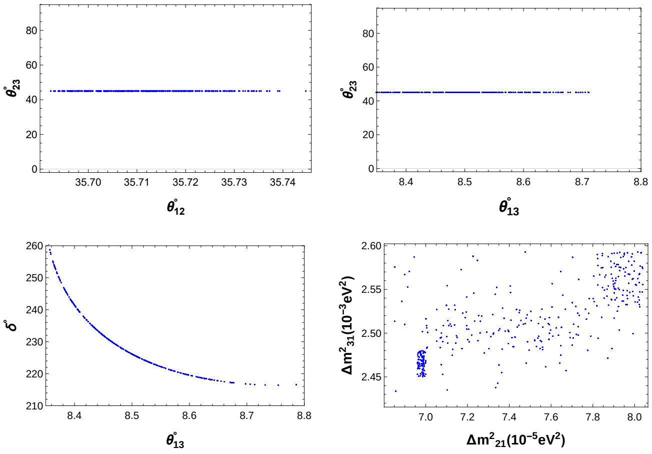

FIGURE 3: Correlation plot for different neutrino oscillation parameters for normal mass hierarchy (NH).

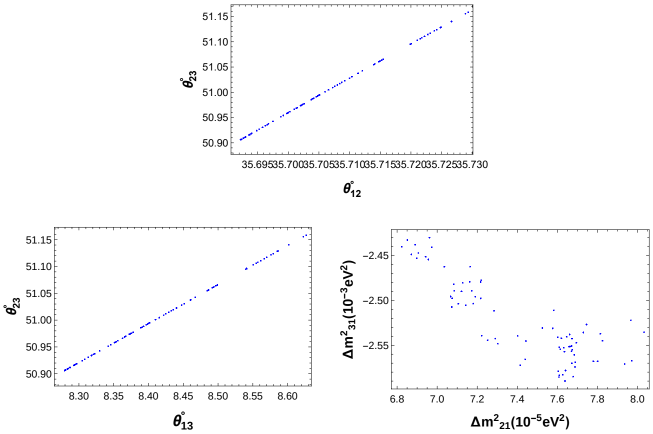

FIGURE 4: Correlation plot for different neutrino oscillation parameters for inverted mass hierarchy (IH).

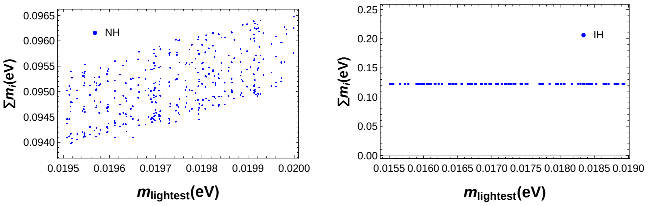

FIGURE 5: Model predictions for the sum of neutrino masses (Σm_i) versus lightest neutrino mass (m_{lightest}) for NH and IH.

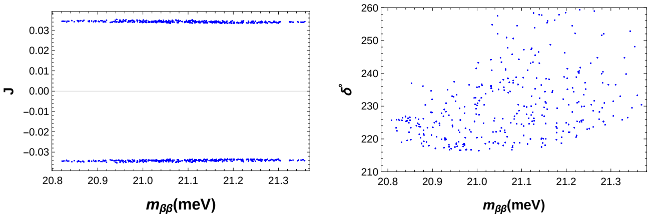

FIGURE 6: Model predictions for Jarlskog invariant (J) versus effective Majorana mass $|m_{\beta\beta}|$ and Dirac CP-violating phase δ versus effective Majorana mass $|m_{\beta\beta}|$ for NH.

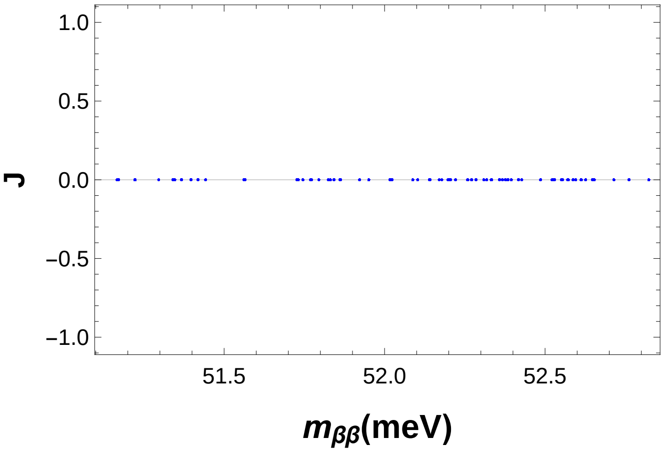

FIGURE 7: Model predictions for Jarlskog invariant (J) versus effective Majorana mass $|m_{\beta\beta}|$ for IH.

4. CONCLUSION

We have presented a minimally modified AF model. The contribution from antisymmetric Dirac mass terms allowed by A_4 only plays a nontrivial role in producing deviation from TBM. Unlike other minimally modified AF models, our model can retain the value of $\theta_{23} = 45^\circ$ while predicting evenly distributed values of Δm^2_{21}, Δm^2_{31}, and θ_{13} within the current 3σ range for NH. Moreover, the values of θ_{12} and δ for NH have more definite predictions. In the case of IH, although the model predicts the values of Δm^2_{21}, Δm^2_{31}, θ_{13}, θ_{23}, and θ_{12} in 3σ range, the predicted value of $\delta = 0^\circ$ shows deviation from the current global fit data. Hence, the model disfavors IH and predicts NH as the preferred pattern for neutrino masses.

CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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