

Bianchi Type V Bulk Viscous String With Electromagnetic Field in Barber's Second Self Creation Theory of Gravitation

Mohini R. Ugale^{a*}, Sagar A. Bhakte^{b†}, Shital B. Deshmukh^{c‡}

^aDepartment of Science and Humanities, Sipna College of Engineering and Technology
Amravati, 444 701 Maharashtra, India

^bDepartment of Mathematics, Prof Ram Meghe college of Engineering and Management,
Badnera, Dist. Amravati 444 701 Maharashtra, India

^cDepartment of Mathematics, Ramdeobaba University Katol Road, Nagpur, 440013 Maharashtra, India

[†]Corresponding Author: sagarbhakte25@gmail.com, Orcid Id 0009-0003-8828-5003

^{*}E-mail: mrugale@sipnaengg.ac.in, Orcid Id 0000-0001-5891-6139

[‡]E-mail: shitalsolanke77@gmail.com, Orcid Id 0000-0002-3176-368X

Abstract: In this article, the Barber's second self-creation theory framework is used to explore the homogeneous and anisotropic Bianchi type-V metric, in reference to a bulk viscous fluid composed of one-dimensional strings embedded in an electromagnetic field. To obtain precise results, we employed power law volumetric expansion of the cosmos, and an extra physically valid situation that gives a trace of the energy-momentum tensor zero. We reviewed the model's kinematics and physical properties, as well as inquiries into the jerk parameter and energy conditions. Additionally, research is done on the effects of the electromagnetic field on the kinematical and physical features of the derived universe.

Keywords: Bulk Viscous Fluid, Cosmic Strings, Electromagnetic Field, Barber's second self-creation theory.

1. Introduction

Recent cosmological findings, such as high redshift SNeIa [1-7], cosmic microwave background radiation [8,9], large-scale structure [10-12], baryon acoustic oscillations [10], and weak lensing [13], support the idea that the universe is accelerating. The cause of the acceleration of the universe is unknown and is commonly referred to as the dark energy (DE) problem, which is due to the negative pressure of the universe. To report this issue, two different approaches have been stood up, one is to develop viable DE models, and another modifies Einstein's general theory of relativity (GTR). Even if we don't know much about DE, it must be explained. Although recent studies have confirmed that the cosmos is currently accelerating, physicists say there is no assurance that this acceleration will continue indefinitely. Furthermore, even if the cosmos may have an oscillating pattern of expansion, some scientists conclude that the current acceleration of the universe may be followed by a deceleration. Cosmologists have put up numerous theories to explain the universe's cosmic acceleration. It is often reported that altered theories of gravity, which are workable substitutes for the GTR, have attracted a lot of attention recently.

A scalar tensor theory of gravity that integrates Mach's Principle within a relativistic framework was developed by Brans and Dicke [14]. Barber [15] offered two ongoing creation hypotheses. The first is a revision of Brans-Dicke theory, while the second adopts GTR to account for continuous matter generation within observational constraints. These altered theories propose that the universe is made up of self-contained gravitational and matter fields. Brans claims that Barber's initial idea is inconsistent overall and does not conflict with experimental findings. In his second self-creation theory, Einstein's GTR is modified to variable G-theory, which is inside the observational limit and predicts local effects. According to his assumption, the gravitational coupling of the Einstein field equations can be a variable scalar on space-time, which links to the energy-momentum tensor's (EMT) trace.

Barber's second hypothesis of self-creation uses the following field equations:

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi\phi^{-1} T_{ij}, \quad (1)$$

And
$$\square \phi = \phi_{;k}^k = \frac{8\pi}{3} \mu T, \quad (2)$$

where μ is an experimentally determined coupling constant. The measurement of the deflection of light limits the value of coupling constant to $|\mu| \leq 10^{-1}$, T_{ij} is the EMT, $G = \phi^{-1}$, T is the trace of the EMT, i.e. $T = T_i^i$ and the other symbols have their usual meaning.

Several investigations have focused on examining cosmological theories within Barber's self-creation frameworks. Aditya et al [16] examined Kaniadakis holographic DE universe of Bianchi type II metric within the framework of self-creation gravity theory. Hadole & Nimkar [17] investigated a comparative study of Bianchi type-VI₀ cosmological models in self-creation theory formulated by Barber under the influence of cosmic string, perfect fluid, thick domain wall, wet dark fluid and macroscopic body. Raut & Rautkar [18] discussed the exact solution of Einstein's field equations for LRS Bianchi type-I cosmological model. Pawar et al [19,20] constructed exact kantowski-sach cosmological models with constant EoS parameter and magnetized anisotropic dark energy models in Barber's second self-creation theory. Pradhan & Vishwakarma [21] considered perfect fluid source for an LRS Bianchi type-I metric in Barber's second self-creation theory and investigated some exact solutions for the vacuum universe, Zel'dovich universe and radiation universe. Rao & Vinutha [22] presented plane symmetric string cosmological models in self-creation theory of gravitation. Hegazy [23] obtained a formula for calculating entropy of the universe in terms of the viscosity and used it to study and compare the Entropy, Enthalpy, Gibbs energy and Helmholtz energy of the universe in the presence of a viscosity term in the self-creation theory of gravitation and in the GTR. Later on, Reddy et al [24] formulated Bianchi type-II bulk viscous string cosmological model in self-creation theory of gravitation. Rao & Sireesha [25] obtained and presented the anisotropic axially symmetric cosmological model in Barber's second self-creation theory of gravitation when the EMT is a bulk viscous fluid containing one-dimensional cosmic strings. A variety of cosmological models [26-32] have been studied by eminent scholars.

Bulk viscosity combined with a string of clouds is an intriguing form of an EMT source. We incorporated such an EMT source into our work, as influenced by the following work. Prasuna et al [33] investigated Bianchi type-III bulk-viscous string cosmological models in the framework of scalar-tensor theory of gravitation. Siddiqi & Al-Dayel [34] examined relativistic bulk viscous fluid string spacetime. Dabre & Makode [35] used the plane symmetric LRS Bianchi type I metric to study bulk viscous fluid coupled to a string of clouds. Daimary & Baruah [36] studied Bulk Viscous Strings within the Framework of Saez-Ballester Theory in Five-Dimensional Spacetime. Kayarkar & Deo [37] investigated Bianchi type VI₀ bulk viscous cosmic string model in modified theory with LVDP. Similarly, several of the authors [38-41] have thoroughly explored string viscous fluids in various circumstances.

The study of magnetic fields is an efficient method for understanding the early stages of cosmic evolution. Singh [42] studied an anisotropic Bianchi-I cosmological model filled with bulk viscous fluid and magnetic field in string cosmology. Gioia & Montani [43] examined cosmological perturbations and gravitational instability of the Bianchi I model with a magnetic field. Sharma & Singh [44] constructed Bianchi type-II string cosmological model with a magnetic field in $f(R, T)$ gravity. Liebscher et al [45] oscillatory singularities in Bianchi models with magnetic fields. Bali & Pareek [46] presented Bianchi type I string dust cosmological models in the presence and absence of magnetic field. Again, Bianchi type V magnetized string dust bulk viscous fluid cosmological model with variable magnetic permeability is investigated by Bali [47]. Sing & Ram [48] discussed the dynamics of spatially homogeneous and anisotropic Bianchi type-III string cosmological model in presence of bulk viscous fluid and electromagnetic field. Bali et al [49] Bianchi type VI₀ magnetized Barotropic Bulk Viscous Fluid Massive String Universe in General Relativity.

The above-mentioned literature motivates us to perform additional research in this field. As a result, the purpose of this work is to identify certain exact solutions in the Bianchi V model with magnetized viscous fluid in string cosmology and examine their implications for the cosmic early and late time development.

The work is divided into distinct sections. Section 2 covers metric and field equations, whereas Section 3 describes how to solve these field equations. Section 4 discusses several essential kinematical parameters. Finally, Section 5 provides a conclusion .

2. Metric & Basic equations

We consider the homogeneous and anisotropic Bianchi type-V metric as

$$ds^2 = dt^2 - A^2 dx^2 - e^{2mx} (B^2 dy^2 + C^2 dz^2), \quad (3)$$

where A , B , and C are the metric potentials considered as functions of cosmic time t only.

A bulk viscous fluid with one-dimensional strings immersed in an electromagnetic field, as an EMT, is calculated as

$$T_{\mu}^{\nu} = (\rho + \bar{p})u_{\mu}u^{\nu} - \bar{p}g_{\mu}^{\nu} - \lambda x_{\mu}x^{\nu} + E_{\mu}^{\nu}, \quad (4)$$

And
$$\bar{p} = p - 3\xi H, \quad (5)$$

where $\rho = \rho_p + \lambda$ is the proper string energy density with particles attached to them and ρ_p is the particle energy density, λ is the string's tension density, $3\xi H$ is bulk viscous pressure, $\xi(t)$ is the coefficient of bulk viscosity, H is Hubble's parameter, x^{ν} denotes a unit space-like vector for the cloud string and u^{ν} denotes a four-velocity vector satisfying the conditions, $u^{\nu}u_{\nu} = 1 = -x^{\nu}x_{\nu}$ and $u_{\nu}x^{\nu} = 0$.

In a co-moving coordinate system, we have

$$u^{\nu} = (0, 0, 0, 1), \quad x^{\nu} = (0, 0, C^{-1}, 0). \quad (6)$$

Also, $E_{\mu\nu}$ is a part of EMT given as

$$E_{\mu\nu} = \frac{1}{4\pi} \left[F_{\mu\alpha}F_{\nu\beta}g^{\alpha\beta} - \frac{1}{4}g_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta} \right], \quad (7)$$

where $F_{\mu\nu}$ is electromagnetic field tensor that satisfy Maxwell equations,

$$F_{[\mu\nu,k]} = 0. \quad (8)$$

In a comoving co-ordinate system, the incident magnetic field is taken along z-axes with the help of Maxwell equation (8), the components of $F_{\mu\nu}$ except F_{12} are vanish i.e. F_{12} is the only non-vanishing component along z-axes.

$$F_{12} = \text{Constant} = M \quad (\text{Say})$$

From equation (7), we have

$$E_1^1 = E_2^2 = \frac{M^2}{8\pi A^4} \quad \text{and} \quad E_3^3 = E_4^4 = -\frac{M^2}{8\pi A^4}. \quad (9)$$

Now, the field equations (1) & (2) for metric (3) using (4) are

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = -8\pi\phi^{-1} \left((p - 3\xi H) - \frac{M^2}{8\pi A^4} \right), \quad (10)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{m^2}{A^2} = -8\pi\phi^{-1} \left((p - 3\xi H) - \frac{M^2}{8\pi A^4} \right), \quad (11)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = -8\pi\phi^{-1} \left((p - 3\xi H) - \lambda + \frac{M^2}{8\pi A^4} \right), \quad (12)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3m^2}{A^2} = 8\pi\phi^{-1} \left(\rho - \frac{M^2}{8\pi A^4} \right), \quad (13)$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0, \quad (14)$$

$$\ddot{\phi} + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \dot{\phi} = \frac{8\pi}{3} \mu (\rho - 3p + \lambda + 9\xi H), \quad (15)$$

where the overhead dot ($\dot{\cdot}$) denotes the derivative with respect to cosmic time t .

Also, we find some kinematical space-time quantities of physical interest in cosmology, as follows:

The spatial volume V are defined as

$$V = ABC. \tag{16}$$

The volumetric expansion rate of the universe is described by the generalised mean Hubble's parameter H given by

$$H = \frac{1}{3} \sum_{i=1}^3 H_i = \frac{1}{3} (H_1 + H_2 + H_3), \tag{17}$$

in which $H_1 = \frac{\dot{A}}{A}$, and $H_2 = H_3 = \frac{\dot{B}}{B}$ denotes the directional Hubble's parameters.

Using (16) and (17), we have obtained the expansion scalar θ , mean anisotropy parameter A_m , shear scalar σ , and deceleration parameter q as

$$\theta = \frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} = 3H, \tag{18}$$

$$A_m = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2, \tag{19}$$

$$\sigma^2 = \frac{1}{2} \left(\sum_{i=1}^3 H_i^2 - \theta^2 \right), \tag{20}$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{d}{dt} \left(\frac{1}{H} \right). \tag{21}$$

3. Solution of Field Equations

Now from (14) we obtain $A^2 = kBC$, where k is an integrating constant. Without loss of generality, we take k as unity, which gives

$$A^2 = BC. \tag{22}$$

The field equations (10) - (15) are a system of five independent equations with seven unknowns $B, C, p, \xi, \lambda, \rho$ and ϕ . To fully solve the aforementioned system, we need two more equations that relate the parameters. Therefore, we examine the following physically realistic conditions:

We take the power law volumetric expansion of the universe as [38]

$$V = t^{3n}, \tag{23}$$

where n is an arbitrary constant.

Solving (10) & (11) with the use of (16), (22) & (23), we obtain,

$$A = t^n, B = t^n e^{\left(\frac{c_1 t^{1-3n} - c_2 - n}{3n-1} \right)}, \text{ and } C = t^n e^{-\left(\frac{c_1 t^{1-3n} - c_2 - n}{3n-1} \right)}. \tag{24}$$

From (24) in (3) we get,

$$ds^2 = dt^2 - t^{2n} dx^2 - t^{2n} e^{2\left(\frac{c_1 t^{1-3n} - c_2 - n}{3n-1} + mx \right)} \left(dy^2 + e^{-4\left(\frac{c_1 t^{1-3n} - c_2 - n}{3n-1} \right)} dz^2 \right). \tag{25}$$

The developed model in (25) appears to be singularity-free till infinite time.

Reddy et al [24], as well as Rao & Prasanthi [50], discussed the condition where the trace of EMT vanishes. So, we utilise an extra physically valid condition, because the field equations are very nonlinear that the trace of EMT is zero, which gives

$$\rho - 3p + \lambda + 9\xi H = 0. \tag{26}$$

From (24) & (26) in (15) we obtain,

$$\phi = c_1 + c_2 t^{1-3n}. \tag{27}$$

From (13), the energy density is

$$\rho = \frac{M^2 t^{2-n} - (3m^2 t^{2(1-n)} + c_1^2 t^{2(1-3n)} - 3n^2)(c_1 t^{3n} + c_2 t)}{8\pi t^{3n+2}}. \tag{28}$$

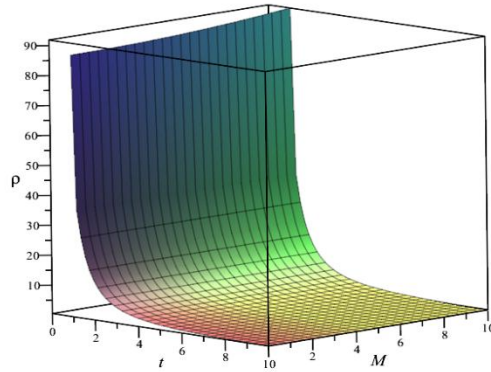


Fig.1: Behaviour of energy density vs. time for $c_1 = 3, c_2 = 5, m = 7, n = 12$.

The fall of energy density is seen from Figure 1 as cosmic time and the effect of magnetic field M increases.

The tension density is

$$\lambda = \frac{M^2 t^{-4n}}{4\pi}. \tag{29}$$

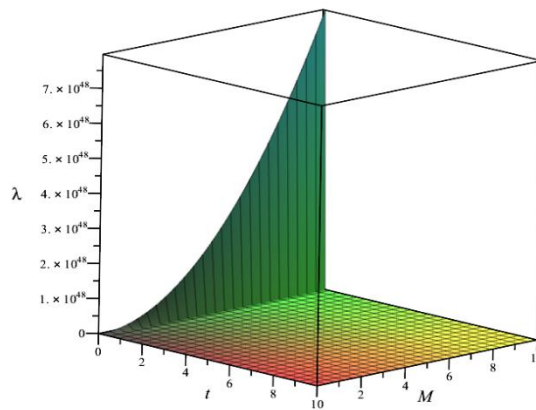


Fig.2: Behaviour of tension density vs. time for $n = 12$.

Figure 2 clearly shows that tension density decreases over time. However, it should be noted that as the effect of the magnetic field increases, tension density decreases from a high value.

The particle density is

$$\rho_p = - \left(\frac{M^2 t^{2-n} + (3m^2 t^{2(1-n)} + c_1^2 t^{2(1-3n)} - 3n^2)(c_1 t^{3n} + c_2 t)}{8\pi t^{3n+2}} \right). \quad (30)$$

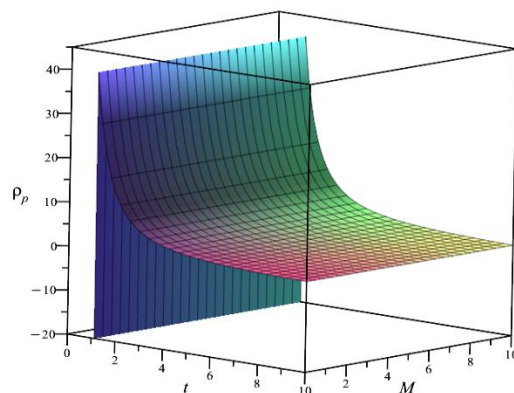


Fig.3: Behaviour of particle density vs. time for $c_1 = 3, c_2 = 5, m = 7, n = 12$.

The particle density appears to shift from negative to positive, as illustrated in Figure 3 with increasing time and magnetic field. Figures 2 and 3 demonstrate that a string-dominated phase is caused by positive tension density ($\lambda > 0$) and a negative particle density ($\rho_p < 0$) when time t is very small. However, as time and magnetic field expand, tension and particle density both decrease in positive, but particle presence is greater than string presence, indicating a particle-dominated era, but both eventually become zero with infinite time, resulting in the disappearance of particles and strings from the universe. These findings are consistent with Singh & Baro [39] and Daimary & Baruah [51].

We assume that the coefficient of viscosity should vary with the expansion scalar in such a way that

$$\xi\theta = \xi_0 = \text{Constant}. \quad (31)$$

Therefore, we obtain

$$\xi = \frac{\xi_0 t}{3n}. \quad (32)$$

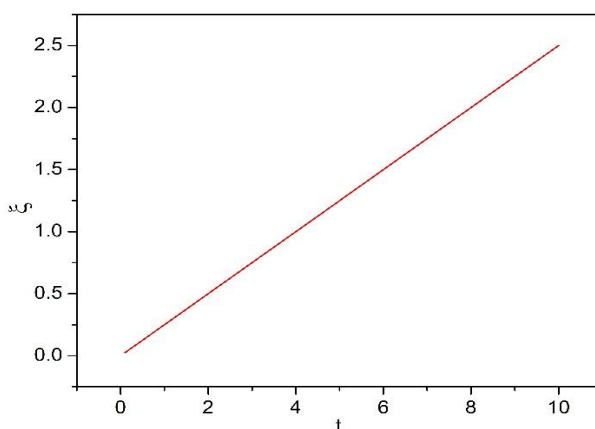


Fig.4: Behaviour of coefficient of bulk viscosity vs. time for $n = 12, \xi_0 = 9$.

The coefficient of bulk viscosity is a stringent exponential function of time and is unaffected by the magnetic field throughout its existence.

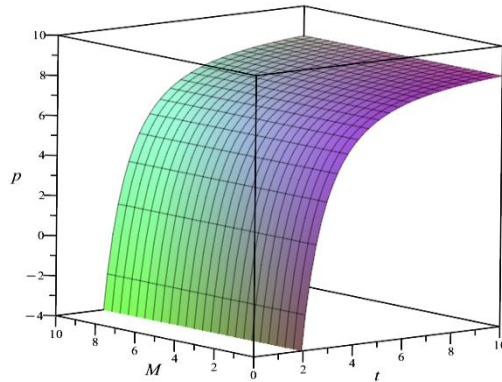


Fig.5: Behaviour of pressure vs. time for $c_1 = 3, c_2 = 5, m = 7, n = 12, \xi_0 = 9$.

From (11), the Pressure is

$$p = \frac{M^2 t^{2-n} + 8\pi\xi_0 t^{3n+2} - \left[c_1^2 t^{2(1-3n)} - m^2 t^{2(1-n)} + n(3n-2) \right] (c_1 t^{3n} + c_2 t)}{8\pi t^{3n+2}}. \quad (33)$$

Pressure appears to be growing over time, from negative to positive, with the steady action of magnetism (Figure 5).

The equation of state (EoS) for string fluid is given by

$$\varepsilon = \frac{\rho}{\lambda} = \frac{M^2 t^{2-n} - \left(3m^2 t^{2(1-n)} + c_1^2 t^{2(1-3n)} - 3n^2 \right) (c_1 t^{3n} + c_2 t)}{2t^{3n+2} M^2 t^{-4n}}. \quad (34)$$

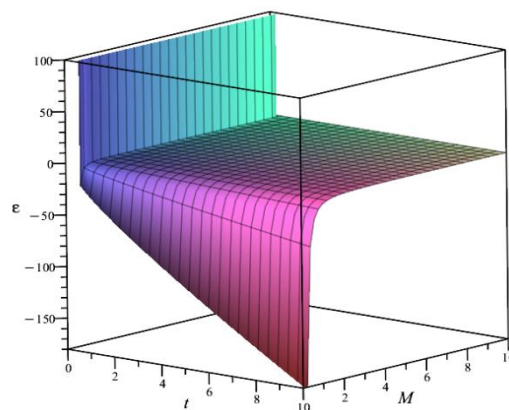


Fig.6: Behaviour of string EoS parameter vs. time for $c_1 = 3, c_2 = 5, m = 7, n = 12$.

The EoS parameter suddenly declines from positive to negative. For tiny magnetism, it drops deeply into the negative, but as the magnetism increases, it becomes steady for the rest transition.

4. Some Important Parameters

The Hubble's parameter & Expansion scalar are

$$H = \frac{n}{t}. \quad (35)$$

$$\theta = \frac{3n}{t}. \quad (36)$$

Figure 7 illustrates how the expansion scalar and the mean Hubble parameter are declining with time. It demonstrates how the universe's expansion slows down over time.

The mean anisotropy parameter

$$A_m = \frac{2c_1^2 t^{2(1-3n)}}{3n^2}. \quad (37)$$

Expression (32) allows us to conclude that the created universe model is purely anisotropic, as the mean anisotropy parameter never approaches 0.

The shear scalar

$$\sigma^2 = c_1^2 t^{-6n}. \quad (38)$$

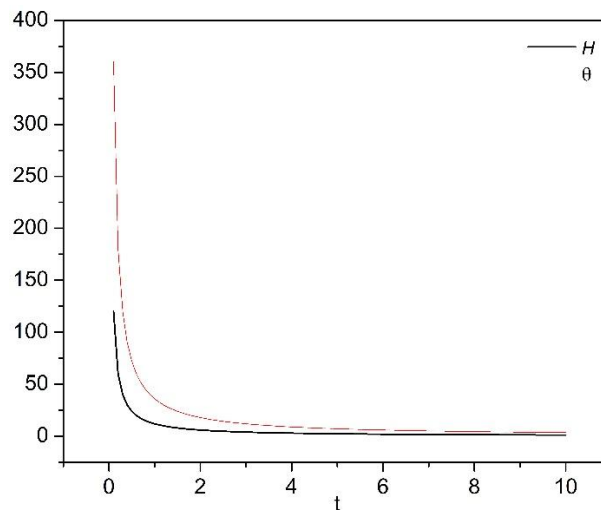


Fig.7: Behaviour of Hubble's parameter vs. time for $n = 12$.

The deceleration parameter

$$q = -1 + \frac{1}{n}. \quad (39)$$

The deceleration parameter appears to be constant in expression (39), yet for a selected choice of constant $q \rightarrow -0.91$ demonstrating an accelerating universe.

4.1 Jerk Parameter

Due to the low density of DE, which overcomes the gravitational pull of matter in the universe, cosmic expansion was initially sluggish. Matter density decreased, and the age of DE dominance began with cosmic expansion. Numerous scholars have explained how the cosmic jerk, which eventually happens for various universe models with a positive jerk parameter and a negative deceleration parameter, causes the cosmos to realign from a decelerating to an accelerating phase.

The jerk parameter is defined and obtained as

$$j(t) = \frac{\ddot{a}}{aH^3} = \frac{n^2 - 3n + 2}{n^2}. \quad (40)$$

In expression (40), we observe that the jerk parameter is a constant function. We know that, if $j = 1$ shows Λ CDM model, $j < 1$ shows a Quintessence type model and $j > 1$ gives phantom type model. For our selection of constant, $n = 12$ we get $j = 0.76 < 1$ shows Quintessence-type universe.

4.2 Energy Conditions

The energy conditions are respectively defined as [52]

- (i) Null Energy Condition : $\rho + p \geq 0$
- (ii) Weak Energy Condition : $\rho \geq 0, \rho + p \geq 0$
- (iii) Strong Energy Condition : $\rho + 3p \geq 0$
- (iv) Dominant Energy Condition : $\rho \pm p \geq 0, \rho \geq 0$

From (28) and (33), we can express the conditions as

$$\rho + p = \frac{M^2 t^{2(1-2n)} + 4\pi\xi_0 t^2 - \left[c_1 \left(m^2 t^{2(1-n)} + c_1^2 t^{2(1-3n)} - n \right) + c_2 \left(m^2 t^{3-5n} + c_1^2 t^{3(1-3n)} - n t^{1-3n} \right) \right]}{4\pi t^2}. \quad (43)$$

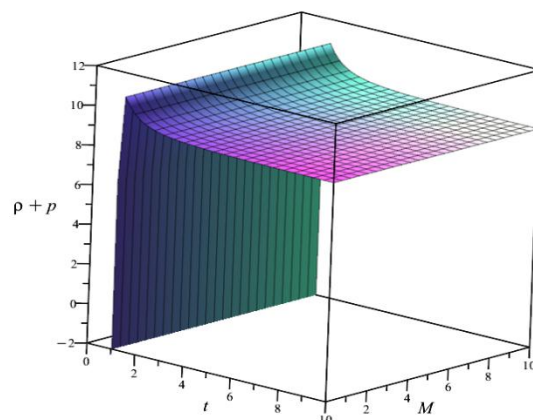


Fig.8: Behaviour of NEC vs. time for $c_1 = 3, c_2 = 5, m = 7, n = 12, \xi_0 = 9$.

From Figure NEC is violated in the beginning since $\rho + p < 0$, however later on $\rho + p > 0$, NEC get satisfied

$$\rho + 3p = \frac{2M^2 t^{2(1-2n)} + 12\pi\xi_0 t^2 - c_1 [2c_1^2 t^{2(1-3n)} + 3n(n-1)] + c_2 (3n(1-n)t^{1-3n} - 2c_1^2 t^{3(1-3n)})}{4\pi t^2}. \quad (44)$$

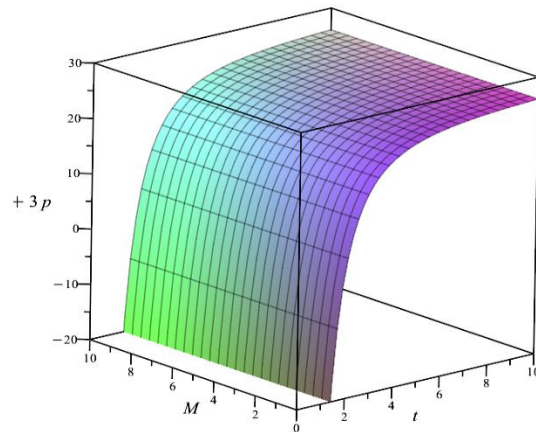


Fig.9: Behaviour of SEC vs. time for $c_1 = 3$, $c_2 = 5$, $m = 7$, $n = 12$, $\xi_0 = 9$.

$$\rho - p = - \left\{ \frac{4\pi\xi_0 t^2 + c_1 [2m^2 t^{2(1-n)} - n(3n-1)] + c_2 [2m^2 t^{3-5n} - n(3n-1)t^{1-3n}]}{4\pi t^2} \right\}. \quad (45)$$

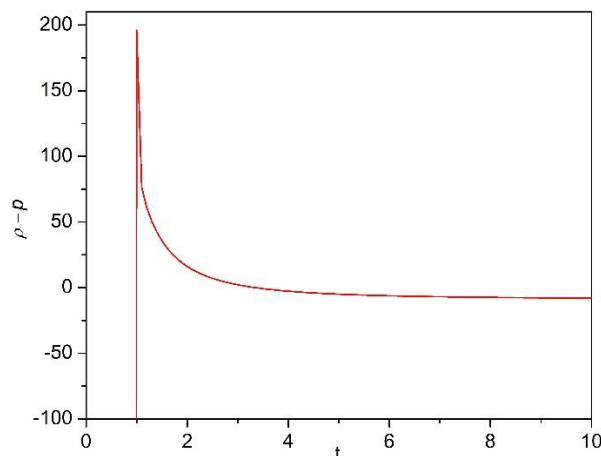


Fig.10: Behaviour of DEC vs. time for $c_1 = 3$, $c_2 = 5$, $m = 7$, $n = 12$.

Since all energy conditions occur in the negative phase, it is evident from Figures 8, 9, and 10 that energy conditions are initially violated. However, everyone eventually becomes satisfied with passing time. Nevertheless, it has been observed that the energy conditions during evolution were unaffected or with a constant effect by electromagnetic field.

5. Conclusions

In the context of Barber's second self-creation framework, this paper investigates the homogeneous and anisotropic Bianchi type V metric in which the EMT is bulk viscous fluid is coupled to a string of clouds with an electromagnetic field. To obtain exact solutions to the field equations, the power law form of volumetric expansion and the condition that gives the trace of EMT zero are applied.

According to the study, magnetic field intensity has a major impact on all physical quantities:

- Energy and tension densities decline over time. The magnetic field has a steady effect on energy density, but it has increased the fall in tension density.
- Particle density changes from negative to positive, transitioning the universe from an early string-dominated phase to a later particle-dominated stage, both of which vanish after infinite times.
- Bulk viscosity decays exponentially over time and is unaffected by magnetism.
- Pressure rises from negative to positive with constant magnetic influence.
- The EoS value rapidly decreases from positive to negative, becoming stable under greater magnetic fields.

Additionally, it is discovered that the created universe is anisotropic, expanding, accelerating, and in a Quintessence-type era with spatial choice of constant values. Magnetic fields have little influence on energy conditions, which are initially violated but eventually satisfied as time passes.

REFERENCES

- [1] A. G. Riess *et al.*, ‘Type Ia Supernova Discoveries at $z > 1$ from the Hubble Space Telescope : Evidence for Past Deceleration and Constraints on Dark Energy Evolution’, *Astrophys J*, vol. 607, no. 2, pp. 665–687, Jun. 2004, doi: 10.1086/383612.
- [2] B. P. Schmidt *et al.*, ‘The High-Z Supernova Search: Measuring Cosmic Deceleration and Global Curvature of the Universe Using Type Ia Supernovae’, *Astrophys J*, vol. 507, no. 1, pp. 46–63, Nov. 1998, doi: 10.1086/306308.
- [3] A. G. Riess *et al.*, ‘Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant’, *Astron J*, vol. 116, no. 3, pp. 1009–1038, Sep. 1998, doi: 10.1086/300499.
- [4] K. Krisciunas *et al.*, ‘ Hubble Space Telescope Observations of Nine High-Redshift ESSENCE Supernovae ’, *Astron J*, vol. 130, no. 6, pp. 2453–2472, Dec. 2005, doi: 10.1086/497640/META.
- [5] D. M. Scolnic *et al.*, ‘The Complete Light-curve Sample of Spectroscopically Confirmed SNe Ia from Pan-STARRS1 and Cosmological Constraints from the Combined Pantheon Sample’, *Astrophys J*, vol. 859, no. 2, p. 101, May 2018, doi: 10.3847/1538-4357/aab9bb.
- [6] A. V. Filippenko and A. G. Riess, ‘Results from the High-z Supernova Search Team’, *Phys Rep*, vol. 307, no. 1–4, pp. 31–44, Dec. 1998, doi: 10.1016/S0370-1573(98)00052-0.
- [7] R. A. Knop *et al.*, ‘New Constraints on Ω_M , Ω_Λ , and w from an Independent Set of 11 High-Redshift Supernovae Observed with the *Hubble Space Telescope*’, *Astrophys J*, vol. 598, no. 1, pp. 102–137, Nov. 2003, doi: 10.1086/378560.
- [8] D. N. Spergel *et al.*, ‘First-Year Wilkinson Microwave Anisotropy Probe WMAP Observations: Determination of Cosmological Parameters’, *Astrophys J Suppl Ser*, vol. 148, no. 1, pp. 175–194, Sep. 2003, doi: 10.1086/377226.
- [9] C. L. Bennett *et al.*, ‘Seven-year wilkinson microwave anisotropy probe (WMAP*) observations: Are there cosmic microwave background anomalies?’, *Astrophysical Journal, Supplement Series*, vol. 192, no. 2, 2011, doi: 10.1088/0067-0049/192/2/17.
- [10] D. J. Eisenstein *et al.*, ‘Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies’, *Astrophys J*, vol. 633, no. 2, pp. 560–574, Nov. 2005, doi: 10.1086/466512.
- [11] M. Tegmark *et al.*, ‘The Three-Dimensional Power Spectrum of Galaxies from the Sloan Digital Sky Survey’, *Astrophys J*, vol. 606, no. 2, pp. 702–740, May 2004, doi: 10.1086/382125.
- [12] M. Tegmark *et al.*, ‘Cosmological parameters from SDSS and WMAP’, *Physical Review D*, vol. 69, no. 10, p. 103501, May 2004, doi: 10.1103/PhysRevD.69.103501.
- [13] B. Jain and A. Taylor, ‘Cross-Correlation Tomography: Measuring Dark Energy Evolution with Weak Lensing’, *Phys Rev Lett*, vol. 91, no. 14, p. 141302, Oct. 2003, doi: 10.1103/PhysRevLett.91.141302.
- [14] C. Brans and R. H. Dicke, ‘Mach’s Principle and a Relativistic Theory of Gravitation’, *Physical Review*, vol. 124, no. 3, pp. 925–935, Nov. 1961, doi: 10.1103/PhysRev.124.925.
- [15] G. A. Barber, ‘On two “self-creation” cosmologies’, *Gen Relativ Gravit*, vol. 14, no. 2, pp. 117–136, Feb. 1982, doi: 10.1007/BF00756918/METRICS.

- [16] M. Dewri and D. Basumatary, 'Anisotropic Bianchi Type VI 0 Dark Energy Cosmological Model for Perfect Fluid Distribution with Electromagnetic Field', *SSRN Electronic Journal*, 2021, doi: 10.2139/ssrn.3977820.
- [17] A. S. Nimkar and S. R. Hadole, 'A Comparative Study of Cosmological Models in Barber Self Creation Theory of Gravitation', *Bulgarian Journal of Physics*, vol. 51, no. 2, Aug. 2024, doi: 10.55318/bgjp.2024.51.2.117.
- [18] V. U. M. Rao and U. Y. D. Prasanthi, 'Bianchi type-V modified holographic Ricci dark energy model in self-creation theory of gravitation', <https://doi.org/10.1139/cjp-2017-0014>, vol. 95, no. 6, pp. 554–558, 2017, doi: 10.1139/CJP-2017-0014.
- [19] J. Daimary and R. Roy Baruah, 'Five Dimensional Bianchi Type-I Anisotropic Cloud String Cosmological Model With Electromagnetic Field in Saez-Ballester Theory', *Frontiers in Astronomy and Space Sciences*, vol. 9, p. 878653, Jun. 2022, doi: 10.3389/FSPAS.2022.878653/BIBTEX.
- [20] D. D. Pawar and Y. S. Solanke, 'Magnetized Anisotropic Dark Energy Models in Barber's Second Self-Creation Theory', *Advances in High Energy Physics*, vol. 2014, pp. 1–9, 2014, doi: 10.1155/2014/859638.
- [21] A. Pradhan and A. K. Vishwakarma, 'LRS BIANCHI TYPE-I COSMOLOGICAL MODELS IN BARBER'S SECOND SELF CREATION THEORY', <https://doi.org/10.1142/S0218271802002207>, vol. 11, no. 8, pp. 1195–1207, Jan. 2012, doi: 10.1142/S0218271802002207.
- [22] V. U. M. Rao and T. Vinutha, 'Plane symmetric string cosmological models in self-creation theory of gravitation', *Astrophysics and Space Science 2009 325:1*, vol. 325, no. 1, pp. 59–62, Oct. 2009, doi: 10.1007/S10509-009-0156-X.
- [23] E. A. Hegazy, 'Bulk Viscous Bianchi Type VI0 Cosmological Model in the Self-creation Theory of Gravitation and in the General Theory of Relativity', *Iranian Journal of Astronomy and Astrophysics*, vol. 6, no. 1, pp. 33–44, Apr. 2019, doi: 10.22128/IJAA.2019.345.1058.
- [24] D. R. K. Reddy, M. P. V. V. Bhaskara Rao, and K. Sobhan Babu, 'Bianchi type-II Bulk viscous string cosmological model in self-creation theory of gravitation', *Astrophysics and Space Science 2014 351:1*, vol. 351, no. 1, pp. 385–389, Feb. 2014, doi: 10.1007/S10509-014-1843-9.
- [25] V. U. M. Rao and K. V. S. Sireesha, 'Axially symmetric string cosmological model with bulk viscosity in self-creation theory of gravitation', *The European Physical Journal Plus 2012 127:5*, vol. 127, no. 5, pp. 1–7, May 2012, doi: 10.1140/EPJP/I2012-12049-3.
- [26] D. R. K. Reddy and R. L. Naidu, 'Kaluza-Klein Cosmological Model in Self-Creation Cosmology', *International Journal of Theoretical Physics 2008 48:1*, vol. 48, no. 1, pp. 10–13, Jun. 2008, doi: 10.1007/S10773-008-9774-2.
- [27] T. Vinutha and K. S. Kavaya, 'Dynamics Of Frw Type Kaluza-Klein Mhrde Cosmological Model In Self-Creation Theory', *Journal of Dynamical Systems and Geometric Theories*, vol. 18, no. 1, pp. 111–129, Jan. 2020, doi: 10.1080/1726037X.2020.1774158.
- [28] E. A. Hegazy and M. E. Kahil, 'Self-Creation Gravity Versus General Relativity: a Cosmological Comparison', *Gravitation and Cosmology 2024 30:3*, vol. 30, no. 3, pp. 254–264, Aug. 2024, doi: 10.1134/S0202289324700166.
- [29] V. U. M. Rao, M. Vijaya Santhi, and T. Vinutha, 'Exact Bianchi type II, VIII and IX string cosmological models in General Relativity and self-creation theory of gravitation', *Astrophysics and Space Science 2008 317:1*, vol. 317, no. 1, pp. 83–88, Jul. 2008, doi: 10.1007/S10509-008-9859-7.
- [30] M. Shen, 'Oscillating cosmological model with a varying Λ term in Barber's second self-creation theory of gravitation', *Astrophysics and Space Science 2016 361:9*, vol. 361, no. 9, pp. 1–4, Aug. 2016, doi: 10.1007/S10509-016-2908-8.
- [31] E. A. Hegazy, 'A New Class of Bianchi Type-I Cosmological Model in Self-Creation Theory', *Iranian Journal of Science and Technology, Transactions A: Science 2019 43:4*, vol. 43, no. 4, pp. 2069–2074, Mar. 2019, doi: 10.1007/S40995-019-00700-W.
- [32] S. D. Katore, R. S. Rane, and K. S. Wankhade, 'FRW Cosmological Models with Bulk-Viscosity in Barber's Second Self-Creation Theory', *International Journal of Theoretical Physics 2009 49:1*, vol. 49, no. 1, pp. 187–193, Nov. 2009, doi: 10.1007/S10773-009-0191-Y.
- [33] Y. Jnana Prasuna, Y. Sobhanbabu, N. L. Pavan Kumar, and V. Srinivas, 'Bianchi type-III bulk-viscous string models in scalar–tensor theory of gravitation', <https://doi.org/10.1142/S0219887824500208>, vol. 21, no. 1, Sep. 2023, doi: 10.1142/S0219887824500208.

-
- [34] R. Bali, A. Pradhan, and H. Amirhashchi, 'Bianchi Type VI 0 Magnetized Barotropic Bulk Viscous Fluid Massive String Universe in General Relativity', *International Journal of Theoretical Physics* 2008 47:10, vol. 47, no. 10, pp. 2594–2604, Mar. 2008, doi: 10.1007/S10773-008-9694-1.
- [35] A. Dabre and P. Makode, 'Viscous Plane Symmetric String Cosmological Model in $f(R)$ Gravity', *Astrophysics*, vol. 67, no. 2, pp. 161–177, Aug. 2024, doi: 10.1007/S10511-024-09826-1/METRICS.
- [36] J. Daimary and R. R. Baruah, 'Anisotropic L. R. S. Bianchi type-V Cosmological Models with Bulk Viscous String within the Framework of Saez-Ballester Theory in Five-Dimensional Spacetime', *Journal of Scientific Research*, vol. 16, no. 1, pp. 115–125, Jan. 2024, doi: 10.3329/JSR.V16I1.65500.
- [37] M. K. Singh and S. Ram, 'Dynamics of Anisotropic Bianchi Type-III Bulk Viscous String Model with Magnetic Field', *International Journal of Theoretical Physics* 2014 53:7, vol. 53, no. 7, pp. 2198–2210, Jan. 2014, doi: 10.1007/S10773-014-2019-7.
- [38] K. Pawar and A. K. Dabre, 'Bulk Viscous String Cosmological Model with Power Law Volumetric Expansion in Teleparallel Gravity', *Astrophysics*, vol. 66, no. 1, pp. 114–126, Mar. 2023, doi: 10.1007/s10511-023-09774-2.
- [39] S. K. Priyokumar and B. Jiten, 'Higher Dimensional LRS Bianchi Type-I String Cosmological Model with Bulk Viscosity in General Relativity', *Indian J Sci Technol*, vol. 14, no. 16, pp. 1239–1249, Apr. 2021, doi: 10.17485/IJST/v14i16.240.
- [40] J. Baro and K. P. Singh, 'String Cosmological Model in 5-Dimensional Space-Time: Interacting with viscous Fluid', *Iranian Journal of Physics Research*, vol. 23, no. 3, pp. 129–133, Nov. 2023, doi: 10.47176/IJPR.23.3.81736.
- [41] A. Gajanan Ingle, P. Shrivastava, R. kumar Dubey, and S. D. Deo, 'Modified Takabayasi String in Bulk Viscous Bianchi Type-III Cosmology: Dynamics in the Presence of Variable Λ ', *Corrosion Management ISSN:1355-5243*, vol. 34, no. 1, pp. 122–145, Aug. 2024, doi: 10.1016/P275CX75.
- [42] C. P. Singh, 'String cosmology with magnetized bulk viscous fluid in Bianchi I universe', *Astrophysics and Space Science* 2012 343:2, vol. 343, no. 2, pp. 773–781, Nov. 2012, doi: 10.1007/S10509-012-1236-X.
- [43] F. Di Gioia and G. Montani, 'Cosmological perturbations and gravitational instability of the Bianchi I model with a magnetic field', pp. 1093–1098, Jul. 2022, doi: 10.1142/9789811258251_0153.
- [44] N. K. Sharma and J. K. Singh, 'Bianchi Type-II String Cosmological Model with Magnetic Field in $f(R, T)$ Gravity', *International Journal of Theoretical Physics*, vol. 53, no. 9, pp. 2912–2922, 2014, doi: 10.1007/s10773-014-2089-6.
- [45] S. Liebscher, A. D. Rendall, and S. B. Tchapnda, 'Oscillatory Singularities in Bianchi Models with Magnetic Fields', *Annales Henri Poincaré* 2012 14:5, vol. 14, no. 5, pp. 1043–1075, Oct. 2012, doi: 10.1007/S00023-012-0207-7.
- [46] R. Bali and U. K. Pareek, 'Bianchi Type I string dust cosmological model with magnetic field in general relativity', *Astrophys Space Sci*, vol. 312, no. 3–4, pp. 305–310, 2007, doi: 10.1007/s10509-007-9692-4.
- [47] R. Bali, 'Bianchi Type V Magnetized String Dust Bulk Viscous Fluid Cosmological Model with Variable Magnetic Permeability', *International Journal of Theoretical Physics* 2008 48:2, vol. 48, no. 2, pp. 476–486, Aug. 2008, doi: 10.1007/S10773-008-9823-X.