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On the Stability of Nuclei Propagating with Momentum Dependent Interactions

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***Abstract*:** The present study deals with the role of momentum dependent interactions (MDI) in heavy-ion collisions using a semi-classical nuclear model. Our goal here is to understand the stability of nuclei propagating under the influence of momentum dependent interactions. For this study, a semi-classical theory in terms of quantum molecular dynamics model (QMD) is employed. The role of momentum dependent interactions is studied by employing static as well as momentum dependent interactions and then studying the time evolution of the phase space of various fragments. Our observations, spanning over entire periodic table, suggest that momentum dependent interactions destabilize the stable nuclei which start emitting nucleons artificially if propagating with momentum dependent interactions. One should, therefore, be careful while studying the multifragmentation with momentum dependent interactions.

***Keywords*:** momentum, interactions, propagating, fragments, multifragmentation.

I. INTRODUCTION

Nuclear physics at low and intermediate energies [1-6] is one of the most extensively studied branches of the physics. After many decades of critical analysis, nuclear physics has reached a moment of critical differentiation: One of its branches separated itself from the traditional nuclear physics to leap into the unexplored space of deconfinement and quark-gluon plasma. Its second branch led to the study of low energy γ- ray spectroscopy. The third branch deals with the study of nuclear collectivity through giant resonances. Lastly, the branch that grew out of the compound nucleus and fission studies, and matured through the low energy deep inelastic scattering, is now exploring the field of intermediate energy heavy-ion reactions. In last two decades, lots of efforts have been made experimentally as well as theoretically to understand the nuclear physics at intermediate energies which ranges between 10 MeV/A and 2 GeV/A.

The main interest to study the low energy nuclear physics or the heavy-ion collisions is to look for the low-density phenomena. As the colliding nuclei cannot compress each other substantially at low incident energies, one has studied the physics of sub-density phenomena that give unique possibility to look for the nuclear interactions, fusion –fission, cluster radioactivity as well as formation of super heavy nuclei.

With the passage of time, one was able to accelerate the heavy–ions with bombarding energies comparable to its rest mass. This opened up new dimensions in the research of nuclear physics. Due to the formation of compressed and hot piece of nuclear matter at intermediate and relativistic energies, it gives unique possibilities to study the properties of nuclear matter at the extreme conditions of temperature and density. Due to the complicated physics, simple pure quantum or classical theories are not adequate to study these reactions. For dynamical calculations, one needs to have a mixture of quantum as well as classical theories generally referred as semi-classical theories. In any realistic theory developed for the heavy ion collisions, nuclear interactions can be represented in terms of real and imaginary parts of the G-matrix. It is worth mentioning that G-matrix is higher version of the reaction matrix where Pauli principle is also incorporated. The real part of the G-matrix represents the nucleon-nucleon potential, where as imaginary part is responsible for the nucleon-nucleon scatterings. The real part of the G-matrix cannot be solved as such [3, 4, 5]. Instead, one parameterizes it in terms of static Skyrme forces supplemented by the Yukawa, Coulomb as well as momentum dependent interactions responsible for the relative velocities of different nucleons. Generally, momentum dependent interactions, due to their nature, generate repulsion in the medium and may destabilize the colliding nuclei [6, 7]. This can have a serious implication if one is studying the multifragmentation, where many body correlations and fluctuations are very sensitive and important. It is worth mentioning that multifragmentation (breaking of nuclei into many pieces) is one of the most sought-after phenomena in heavy ion physics. We shall here concentrate on the effect of momentum dependent interactions on the stability of different nuclei and shall compare the outcome with static interactions. This will give us opportunity to see whether nuclei can be kept cold and stable or not. Section II deals with the models and section III deals with results and discussion followed by conclusion in Section IV.

II. MODEL

A: QUANTUM MOLECULAR DYNAMICS (QMD) MODEL

In QMD model nucleons are represented by Gaussian wave packets which interact via mutual two and three-body interactions [3, 7-17]. Each nucleon in nuclear matter propagates under the classical equation of motion. It is widely accepted that the static equation of state cannot describes the heavy-ion reaction adequately. The fate of a reaction depends not only on the density, but also on the momentum space. In the frame work of G matrix, which is a solution of Bethe-Goldstone equation, the momentum dependence comes in a natural way [15]. However, one cannot solve the Bethe-Goldstone equation numerically at each point in the phase space and time. Therefore, the only way is to parameterize the density and momentum-dependent G matrix elements in terms of two-body interactions. However, the numerical utility of the G matrix is limited. As an alternate, we use parameterized momentum-dependent potential which takes care of the momentum dependence in mean field potential.

The momentum dependent interactions are obtained by parameterizing the measured energy dependence of the proton-nucleus optical potential [16]. A parameterized form of the local plus momentum dependent potential can be represented as:

$$U(\rho)=\alpha\left(\frac{\rho}{\rho_0}\right)+\beta\left(\frac{\rho}{\rho_0}\right)^{\gamma}+\delta.\ln^2\left[\varepsilon\left(\rho/\rho_0\right)^{2/3}+1\right]\rho/\rho_0 .$$

In table I, we list different parameters used in the static as well as momentum dependent interactions.

Table I: The parameters of static and momentum-dependent interactions.

K(MeV)	α(MeV)	β(MeV)	γ	δ(MeV)	ε	EOS
200	-356	303	1.17	-----	-----	S
380	-124	70.5	2	-----	-----	H
200	-390(3189)*	320(3176)*	1.14(1.011)*	1.57	21.54	SMD
380	-130(63.13)*	59(49.42)*	2.09(2.12)*	1.57	21.54	HMD

*These values are based on experimental data of Hama et al. [16] fitted by Hartnack and Aichelin [16].

With the help of above methodology, we simulate the reaction on an event-by-event method. This reaction is divided into large number of time steps which takes care of propagation and scattering part. As an outcome, we store the phase-space of all nucleons at all time steps. Generally, reactions are simulated till a time of 200 fm/c. This reaction time is assumed to be large enough for nucleon-nucleon interactions to happen. Now, experimentally, this phase-space cannot be measured, instead, this phase-space is used to further analyze the reactions in terms of some measurable quantities. One of the quantities of recent interest is the size and number of clusters/fragments and their velocity profiles. One, therefore, needs to clusterize the phase-space to extract emitted fragments. There are several different methods to clusterize the phase-space. Among all these methods, minimum spanning tree (MST) method has been used very frequently to study the fragmentation.

B: MINIMUM SPANNING TREE (MST) METHOD

In MST method, two nucleons share the same fragment if their centroids ($\mathbf{r}_i$, $\mathbf{r}_j$) are closer than a distance d_{min},

$$\left|\mathbf{r}_i - \mathbf{r}_j\right| \leq d_{min} \tag{10}$$

where $\mathbf{r}_i$ and $\mathbf{r}_j$ are the spatial positions of both nucleons. The value of d_{min} can vary between 2-4 fm. As reported, the variation of d_{min} has small effect on the multifragmentation [18-20]. For the present study, we take d_{min} = 4 fm. It is worth mentioning that this method can only be used to analyze the asymptotic configurations in which fragmenting system can be viewed as a very dilute mixture of free particles and almost equilibrated fragments. Overlapping of nucleons at asymptotic times around 150-200 fm/c defines directly the clusters [3, 7].

III. RESULTS AND DISCUSSION

For the present study, we restrict ourselves to the dynamics of a single excited projectile, propagating with finite energy of 100 MeV/A in lab frame. Propagation of a single excited projectile was simulated for static soft (S) equation of state and soft equation of state with momentum dependent potential (SMD). As a first part of the problem, we tested few events with reference to technical parameters and made sure that no artificial events are there in the test run. After that we employed quantum molecular dynamics model for generating the phase space of nuclei under study. We have tested through the impact parameter of b = 100 fm which is enough to derive the physics of a single nucleus. The QMD model after very complicated calculations gives us the phase space of nucleons in terms of x, y, z, P_x, P_y and P_z. This phase space was then clusterized using minimum spanning tree (MST) algorithm as described in section II. It is worth mentioning that a large number of algorithms have been reported in the literature for clusterization of phase space [17-20]. The MST method is the simplest among all these methods.

In fig. I, we plot the characteristic emission of fragments and clusters from a single gold nucleus. The left panel is for the soft EoS whereas right panel is for the SMD EoS. We see that the largest fragment $<A^{max}>$ is close to around mass of 197 units with soft EoS where as it is reduced to about 180 units when momentum dependent interactions are also used. As a result, we see not only the nucleons, but also the light charged particles LCP's [2≤A≤4]. This number is nearly insignificant with soft EoS whereas it turns sizeable with SMD EoS. This trend is further supported by the enhanced emission of fragments with mass A=2, medium mass fragments MMF's [5≤A≤9] and intermediate mass fragments IMF's [5≤A≤65] as shown in fig. II. We see that the above conclusions remain valid in these cases also. This clearly indicates that the gold nucleus when propagating under the influence of momentum dependent interactions results in destabilization and as a result it emits nucleons as well as clusters artificially.

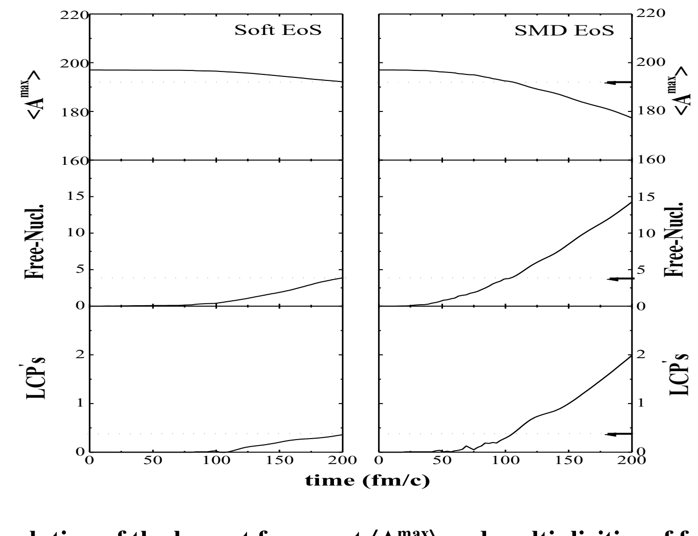

Figure I. The time evolution of the largest fragment ⟨A^{max}⟩ and multiplicities of free nucleons as well as light charged particles, LCPs [2≤A≤4].

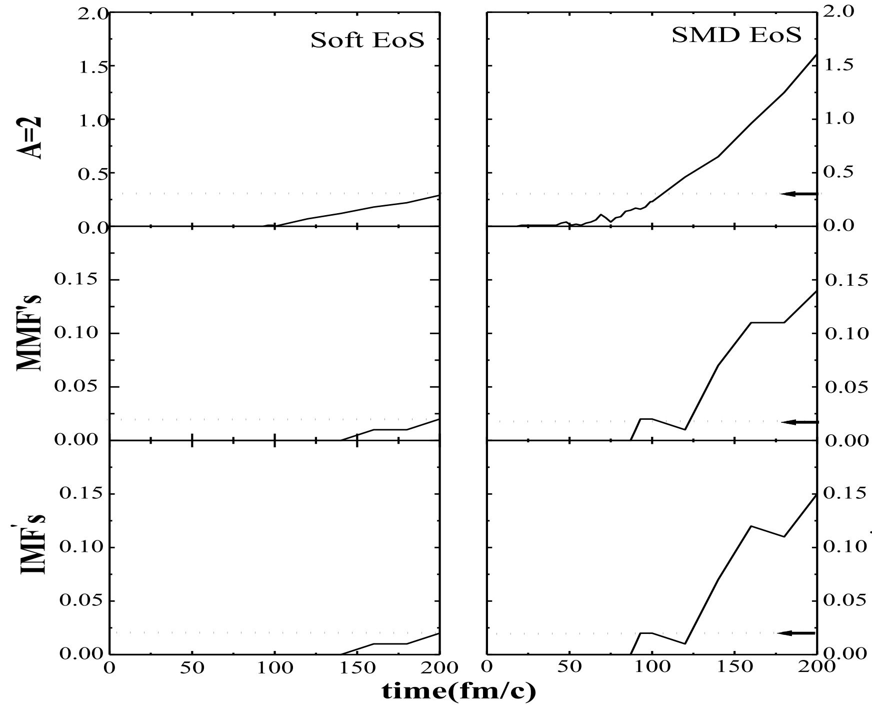

Figure II. The time evolution of fragments with A=2, MMF's [5≤A≤9] and IMF's [5≤A≤65].

This observation is further strengthening when we compare root mean square (rms) radii of nuclei propagating under the influence of soft and SMD EoS. In fig. III, we show the rms radii of ^{40}Ca and ^{197}Au nuclei propagating with soft and SMD equations of states. As we see, SMD EoS destabilizes the nucleus over the time span of 150-200 fm/c effectively whereas soft EoS keeps the most of nucleons bound in nuclei for reasonable reaction time.

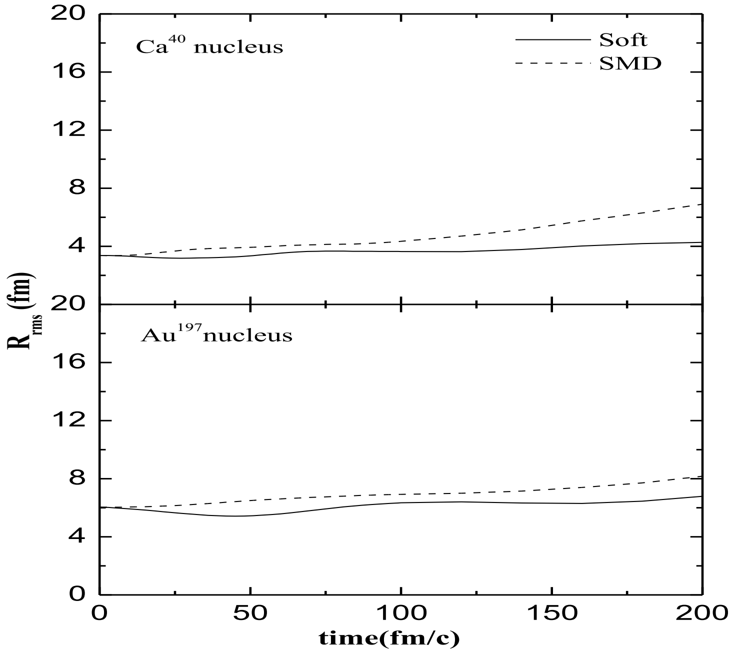

Figure III. The root mean square (rms) radii of ^{40}Ca (upper panel) and ^{197}Au (lower panel) nuclei as a function of reaction time. The solid and dashed lines show the results for static soft and momentum dependent soft equations of state, respectively.

It would be of further interest to extend this study to different nuclei over the periodic table. This also may be helpful to search for the finite size effects in disassembly of nuclear matter. For this, we simulated the nuclei of ^{40}Ca, ^{93}Nb, ^{129}Xe, ^{139}La, ^{167}Er and ^{197}Au for 200fm/c. In figure IV, we display the average size of largest fragment $\langle A^{max} \rangle$ for nuclei ranging between mass 40 and 197. We can see that MDI leads to smaller $\langle A^{max} \rangle$ compared to soft EoS. This decrease is larger in heavy mass region. In other words, effect of MDI is much larger for heavier nuclei compared to light nuclei. This is understandable since heavy nuclei generate higher densities leading to more repulsion. The above observation points toward the importance of finite size effects in the stability of excited nuclei. The larger effects of MDI in the heavier mass region are also pronounced in the multiplicity of various fragments displayed in fig. V.

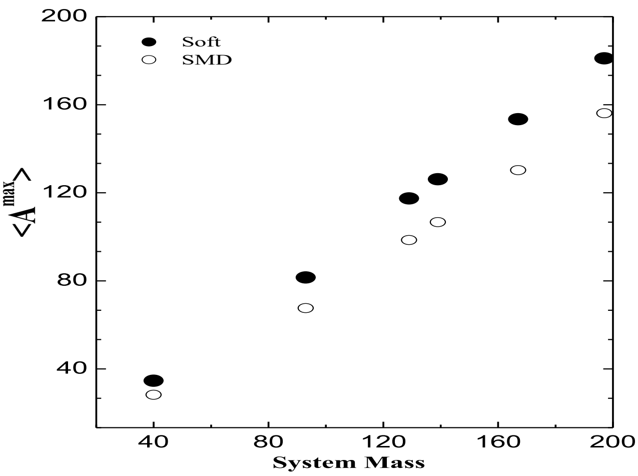

Figure IV. The average size of largest fragment $\langle A^{max} \rangle$ as a function of the mass of system. Here we considered ^{40}Ca, ^{93}Nb, ^{129}Xe, ^{139}La, ^{167}Er and ^{197}Au nuclei.

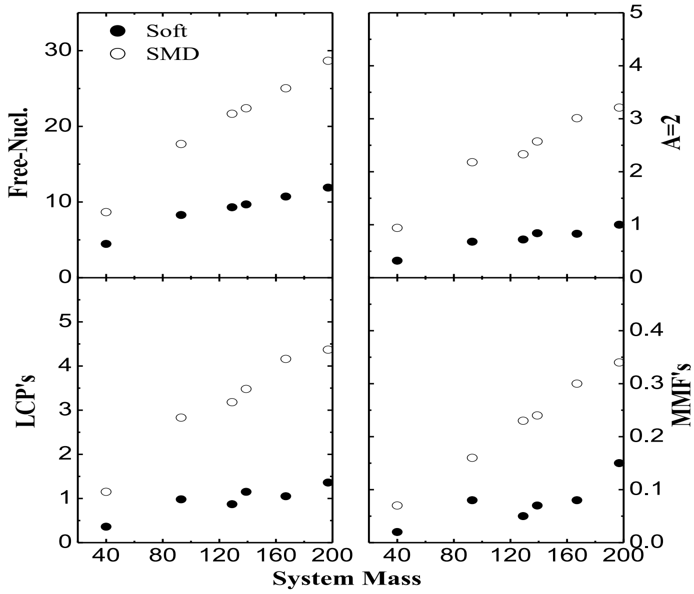

Figure V. The multiplicities of free-nucleons, fragments with mass A=2, light charged particles LCP's [2≤A≤4], medium mass fragments MMF's [5≤A≤9] for ^{40}Ca, ^{93}Nb, ^{129}Xe, ^{139}La, ^{167}Er and ^{197}Au nuclei.

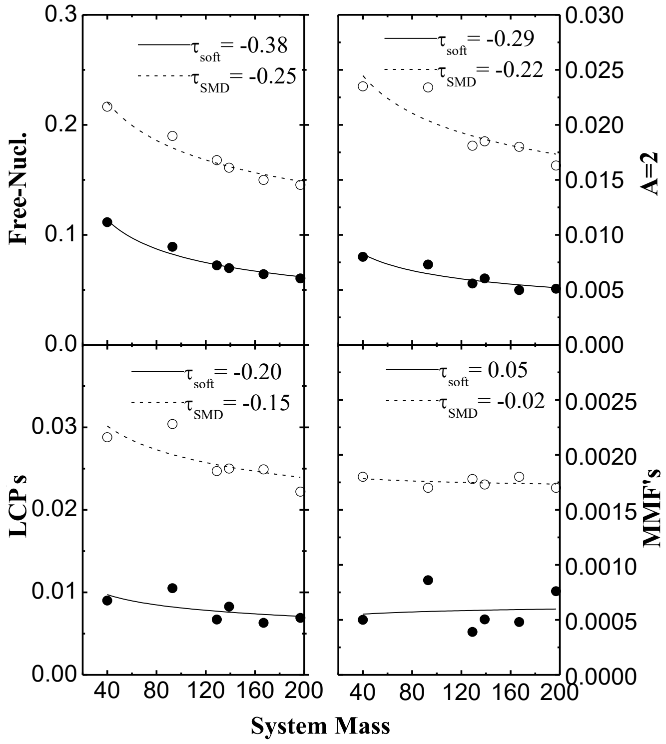

Figure VI. The least square fit of the reduced multiplicities of free-nucleons, fragments with A=2, light mass fragments (LCP's) [2≤A≤4], medium mass fragments (MMF's) [5≤A≤9] as a function of mass for the ^{40}Ca, ^{93}Nb, ^{129}Xe, ^{139}La, ^{167}Er and ^{197}Au systems with static soft equation of state (shown by solid points) and SMD (shown by open circles).

In fig. VI, we again observed the mass dependence phenomenon for reduced multiplicities of various fragments in de-excitation of projectile nuclei. The solid circles represent the reduced multiplicities with soft EoS while open circles show the reduced multiplicities with SMD EoS. We also attempted to fit the reduced multiplicities of various fragments as a function of initial mass of the projectile A with power law $y = cA^{\tau}$ and obtained the values of exponent τ, which are also indicated in the figure. The trend of larger reduced multiplicities obtained with MDI is clearly visible over the entire mass region. The negative slope of power law fit can be attributed to smaller surface-to-volume ratio in the heavier nuclides. As a result, yield of nucleons emitted from the surface decrease with the size of the projectile nucleus. This has clear bearing in the study of fragmentation in heavy-ion collisions.

IV. CONCLUSION

Using the quantum molecular dynamics (QMD) model coupled with minimum spanning tree method, we studied the stability of nuclei propagating under the influence of momentum dependent interactions. We see that the nuclei propagating under the influence of momentum dependent interactions tend to be more unstable compared to the soft equation of state. This has been checked for a large number of masses of nuclei ranging from ^{40}Ca to ^{197}Au. Our study clearly indicates that one should be very careful when studying multifragmentation with momentum dependent interactions.

REFERENCES

1. R. K. Puri, E. Lehmann, A. Faessler and S. W. Huang, *Z. Phys. A* **351**, 59 (1995).
2. S. Kailas, *Parmana J. Phys*. **57, 75** (2001).
3. J. Aichelin and H. Stöcker, *Phys. Lett. B* **176**, 14 (1986), Ch. Hartnack, R. K. Puri, J. Aichelin, J. Konopka, S. A. Bass, H. Stöcker, W. Greiner, *Eur. Phys. J*. **1**, 151 (1998).
4. Bohnet, N. Ohtsuka, J. Aichelin, R. Linden and A. Faessler, *Nucl. Phys. A* **494**, 349 (1989).
5. D. T. Khoa et al., *Nucl. Phys. A* **542**, 671 (1992), D. T. Khoa et al., *Nucl. Phys. A* **529**,

363 (1991), G. Q. Li et al., *Nucl. Phys. A* **534**, 697 (1991).

6. G. Peilert et al., *Phys. Rev. C* **39**, 1402 (1989).
7. J. Aichelin, *Phys. Rep.* **202**, 233 (1991).
8. G. Peilert, A. Rosenhauer, J. Aichelin, H. Stöcker and W. Greiner, *Mod. Phys. Lett. A* **3**, 459 (1998).
9. L. Neise et al., *Nucl. Phys. A* **519**, 375c (1990), M. Berenguer et al., *J. Phys. G*: **18**, 655 (1992).

A. Bohnet et al., *Phys. Rev. C* **44**, 2111 (1991).

10. J. Cugnon, *Phys. Rev. C* **22**, 1885 (1980), J. Cugnon, D. Kinet and J. Vandermeulen, *Nucl. Phys. A* **379**, 553 (1982), J. Cugnon and D. L. Hote, *Phys. Lett. B* **149**, 35 (1984), J. Cugnon and C. Volant, *Z. Phys. A* **334**, 435 (1989), H. Duarte, *Phys. Rev. C* **75**, 024611 (2001).
11. L. Zhuxia, C. Hartnack, H. Stöcker and W. Greiner, *Phys. Rev. C* **44**, 824 (1991).
12. R. K. Puri and S. Kumar, *Phys. Rev. C* **57**, 2744 (1998), J. Singh, S. Kumar and R. K. Puri, *Phys. Rev. C* **62**, 044617 (2000), ibid **65**, 024602 (2002).
13. S. W. Huang, Ph. D. Thesis, University of Tubingen, Germany (1994), and references therein.
14. J. Aichelin, A. Rosenhauer, G. Peilert, H. Stöcker and W. Greiner, *Phys. Rev. Lett.* **58**, 1926 (1987).
15. S. Hama, B. C. Clark, E. D. Cooper, H. S. Sherif and R. L. Mercer, *Phys. Rev. C* **41**, 2737 (1990), C. Hartnack and J. Aichelin, *Phys. Rev. C* **49**, 2801 (1994).
16. J. Singh and R. K. Puri, *J. Phys. G: Nucl. Part. Phys*. **27,** 2091 (2001).
17. S. Kumar and R. K. Puri, *Phys. Rev. C* **58**, 2858 (1998).
18. J. Singh, S. Kumar and R. K. Puri, *Phys. Rev. C* **63**, 054603 (2001).
19. S. Kumar and R. K. Puri, *Phys. Rev. C* **60**, 054607 (1999).